Physics 114 Practice Test 2 (with answers at end!) 2021

Info about the test:

1) It is **open book**/notes/our Moodle site/our Google Drive with solutions.
2) **Please study for it**. Incentivizing studying is the only good reason I can think of for giving a test!
3) You can schedule taking it in any 3 1/2 hour block of time that is convenient for you during the exam period from May 13 - May 20.
4) A Google Doc will be available soon, for you to specify the time you want to take the test. **Thank you for, before May 13, committing to a time that works for you**.
5) **I'll email the test to you at your start time**. You have 3 1/2 hours to email answers back to me. I'll do my best to write a test that, if one has studied, takes comfortably less than 3 hours to do. I'm adding an extra half hour for you to check work, scan, and be sure the scans you email back are legible.
6) If you have an extra time accommodation, that extra time will be added on.

---------------------------Practice test starts here-------------------------------------

Instructions:

- Please write as **legibly** as you can so that your scans that you email back to Amy are highly readable.
- There are 7 questions on this test. Please **choose 5 of them to do for credit.**
- Submit to me **only the 5 solutions** that you want me to evaluate.
(To be fair to everyone, if you submit more I will only look at the first 5.)
- Please show all reasoning and work leading to your answer. **Credit will strongly emphasize your process and your work** ... the way you explain the problem and your journey toward the answer ... not just the answer itself.
- **You have 3 1/2 hours** to email your work back to Amy.
- **Good luck!!!**

Useful data and conversion factors:

Ideal gas constant $R = 8.31$ J/mol · K.
Boltzmann's constant: k = 1.38×10^{-23} J/K
Planck's constant: h = 6.63×10^{-34} J s
Proton mass: 1.66×10^{-27} kg
Electron mass: 9.1×10^{-31} kg
1 eV = 1.60×10^{-19} Joule
Absolute zero: 0K = -273.15 ºC
speed of light: c = 3.00×10^{8} m/s

1. Near the triple point of a substance:
- the liquid-gas coexistence curve has $P(T) = Ae^{-B/T}$
- the solid-gas coexistence has $P(T) = Ce^{-D/T}$

In the expressions above, *A, B, C* and *D* are constants
Please find the triple point temperature.

2. A system of N electrons (which have their usual mass and spin 1/2) exist in a nanowire of length L, which we can model as a one-dimensional box. For this problem, ignore interactions between electrons, treating them like a 1d ideal fermionic gas. What is the Fermi temperature, T_F , in degrees K, if $L = 10^{-6}\ m$ and $N = 10^{8}$?

3. Photons are massless bosons with two polarization states and $\mu = 0$. For a box of volume V and temperature T, please derive the mean number of photons in the box, N .

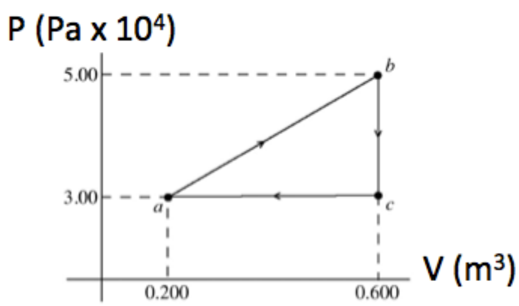
4. Consider a paramagnetic system of N spins that can take on, not two values, but *three* values: $s_i = -1, 0, 1$. The spins sit on a lattice. They do not interact with each other, but they do interact with a magnetic field so $E_i = -\mu B s_i$ is the energy of the i^th^ spin. (Here μ is a magnetic moment.) What are the free energy, F, and the average magnetization $<M>$, of a system of these spins at temperature T? (Hint: Does your $<M>$ come out to have reasonable limits for low and high T?)

5. The function $U(S,V,N)$ is a "master function": all thermodynamic quantities of interest can be obtained by taking suitable derivatives of $U(S,V,N)$. For the variables T,V,N the master function is the Helmholtz free energy $F(T,V,N)$. While energy $U(T,V,N)$ is a valid function, it has lost its "master function" status. Demonstrate this by considering two different substances, A and B, with free energies:

$$F_A(T,V,N) \text{ and } F_B(T,V,N) = F_A(T,V,N) + aTV^2/N$$ where a is a constant.

Show that these two substances have identical energies $U(T,V,N)$ but different equations of state $P(T,V,N)$.

6. The PV diagram below is for an ideal gas, working in a cycle from $a \to b \to c \to a$.



Is the net heat, Q, absorbed by the gas positive, negative or zero? If nonzero, find the net heat, Q.

7. Two friends, Jay and Kay, challenge each other to a 1d random walk contest to see who will get further from their starting point at $x=0$. Jay takes steps to the right and left with an equal probability of ½, and takes $2N$ steps of size a. Kay takes steps to the right with probability ¾, and left with probability ¼, and takes N steps of size a.

For each person, Jay and Kay, please calculate

a) Their mean displacement $<x>$ from the origin

b) Their *rms* displacement $\sqrt{< x^2 - < x >^2 >}$ from the origin

c) Comment on which quantity is bigger for which friend (or if both are the same?) and do a reality-check of why this makes sense to you.

Answers:

1. $T_{triple} = \dfrac{B - D}{\ln A - \ln C}$

2. $T_F = 1.1 \times 10^{13}$ K

(Some explanation: You need to use the 1d density of states for particles in a box, and integrate over all k states up to k_F and set this integral equal to N. You should get $k_F = N\pi / 2L$ where the "2" in denom comes because there will be 2 electronic spin states occupying each k state. Then you use the relationship between k_F and T_F and plug in numbers.)

3. Your derivation hopefully leads to: $N = 2.404 \dfrac{V(kT)^3}{\pi^2 c^3 \hbar^3}$

4. $F = -NkT \ln[1 + 2\cosh(\mu B/kT)]$, $\langle M\rangle = 2N\mu \sinh(\mu B/kT) / [1 + 2\cosh(\mu B/kT)]$.

5. Both U_A and U_B come out to be $F_A - T\left(\dfrac{\partial F_A}{\partial T}\right)_{V,N}$. This comes from the definition $U = F + TS$ and obtaining S as a partial derivative of $F(T,V,N)$. If you look for $P(T,V,N)$ in a similar way as the appropriate partial derivative of F, with the appropriate quantities held constant, you find $P_B = P_A - 2aVT/N$.

6. Positive heat is absorbed (thus, positive work is done by the gas on the outside world). $Q = 4000\ J$

7.

a) <x> for Jay is 0 while for Kay it is $\frac{N}{2}a$

b) x_{rms} for Jay is $\sqrt{2N}a$ while for Kay it is $\sqrt{\frac{3}{4}N}a$.

c) Jay does an "unbiased" walk with $p = q$. So by symmetry, their mean displacement is zero. Kay's walk is biased to the right so it makes sense they move to the right, with nonzero mean displacement of $(p-q)Na$, given N is the number of steps of size a that Kay takes. On the other hand, x_{rms} is proportional to the width of the underlying binomial distribution. This is proportional to pq and to the number of trials. Here, pq is larger for Jay (1/4 vs 3/16) and also Jay does a larger number of trials ($2N$ for Jay vs. N for Kay). Thus, while Kay moves further as measured by the mean displacement, Jay moves further as measured by the *rms* displacement.