Physics 114 Statistical Mechanics Spring 2021
Week 11 Reading and Problem Assignment

This is our second week studying *Fermi-Dirac* or *Bose-Einstein* particles in stat mech. Last week, we began with the general formalism: counting states of the system, finding free energy $\Omega = \Sigma_k \Omega_k$, and the occupation number $\bar{n}_k$ for states labelled with wave number $\vec{k}$. These led us, in a familiar way, to calculating statistical properties of interest.

This week, we apply our understanding of boson statistics to the *Bose Einstein Condensate (BEC)* . We also focus on fermions, half-integer spin particles, no two of which can occupy a single quantum state. The *ideal Fermi gas* is a starting place from which to model electrons moving in a solid, or the degeneracy pressure that keeps a neutron star from collapsing. A key idea is the *Fermi level* ... the highest energy level which N fermions fill at $T = 0^*$. We will look at this ground-state situation, and also look at higher temperatures, with particles excited beyond the Fermi level ... for example, the electrons which carry current in a conductor.

*For a warmup last week you did Schroeder Problem 7.10. There you filled energy levels separated by a constant energy spacing Δ. You hopefully saw that for 5 fermions, the lowest energy state filled levels 0, 1, 2, 3, 4. So the "fermi energy" of this system is $\epsilon_F = 10\Delta$.

Suggested Reading:

G&T Sections

- 6.8, 6.10
- 6.11.2

B&B Sections

- 30.2 - 30.4

Schroeder Sections

- 7.3, 7.6

Warmup Problems:

1: Counting fermions: Schroeder Problem 7.16

2: The Fermi level: Each atom in a piece of copper contributes one conduction electron. Look up the mass density and atomic mass of copper to find Fermi energy, Fermi temperature, and degeneracy pressure. Is room temperature low enough to treat the electrons in copper as a degenerate Fermi gas?

Problems to discuss in our meeting

Note: The * means that these problems are to be handed in. They are due the day after we meet.

1*: Bose-Einstein critical temperature: Schroeder problem 7.65

2: Bose-Einstein Condensate with numbers:
Schroeder problem 7.66 parts (a) - (c)
Note: You'll have an integral you could just do with Wolfram, python, etc. But please look at B&B Appendices C.4 and C.5 and tell us what that integral is "exactly" terms of the Zeta function $\zeta(n)$ and Polylogarithm function $Li_n(z)$.

3: Bose-Einstein Condensate and pressure: G&T problem 6.40

4: High temperature equation of state for quantum gasses
i) Fermi gas: G&T Problem 6.60 parts (a) and (b)

ii) Boson gas: If we had infinite time, I'd suggest doing G&T Problem 6.61. Not possible! So please just compare what you found in i), PV for fermions as given by Eq. (6.268b), to the bosonic expression for PV in Eq. (6.270b). Say how each of these compare to the ideal gas law. What is different about PV for fermions vs. bosons? Wave your hands about why this makes sense, given what you know about the statistics of fermions and bosons.

5*: A neutron star: G&T problem 6.55

6: A fermi gas at low temperature ($T < T_F$):
G&T problem 6.31

7: Chemical potential for a Fermi gas: Schroeder problem 7.32

8*: Toy system of Fermions: G&T Problem 6.57