Physics 114 Statistical Mechanics Spring 2021
Week 10 Reading and Problem Assignment

Overview:

We have done lots of work on the ideal gas. It is "semiclassical" in the sense that we count states in a box using quantum ideas, but then we assume it is dilute enough that every quantum volume λ_{th}^3 contains at most 1 particle. This week, we take on *quantum gasses*. They are noninteracting, so we want to call them "ideal", but are dense enough so *Fermi-Dirac* or *Bose-Einstein* statistics are needed to count how many particles occupy a quantum state indexed by the wave number $\vec{k}$. This is a challenging topic, with many interesting applications, and we break it up into two weeks. This week, we begin with the general idea of how to count the expected number of fermions or bosons, $\bar{n}_k$ in any state $\vec{k}$. To find thermo averages, say energy $\bar{E}$, we must integrate the product $E(\vec{k}) \times g(\vec{k}) \times \bar{n}_k$, where $g(\vec{k})$ is the density of quantum states which we we first met, in the case of an ideal gas, a few weeks ago.

Other quantities we've met, which become exceptionally useful, are grand canonical ensemble and its grand potential. We also find that *activity* which is also known as *fugacity*, $z = f = e^{\beta\mu}$, is an elegant way to parametrize the state of a bose or fermi ideal gas. New pieces of mathematics this week are the *polylogarithm* and *zeta function*, discussed in Appendices of B&B. While this week we learn the nuts and bolts of dealing with bosons and fermions, we bias our work toward the boson gas. (We will save the topic of Bose-Einstein condensation for next week though.) Bosons are particles with integer spins, and one can have an infinite number of these in any quantum state. Photons are massless bosons, and a very important application this week is photons in thermal equilibrium, also known as *black body radiation*. Another important application is quantized vibrations, or *phonons* in solids. We do the *Einstein solid* (familiar to us) and the *Debye solid* (new).

Suggested Reading:

Schroeder sections

- Sections 7.2, 7.4 and 7.5

G&T Sections

- 6.3-6.5.1 *this overlaps with reading we did for Seminar 8*
- Sections 6.7 and 6.9

B&B Sections

- Chs. 23, 24 and 29
- Section 30.1
- Appendices C4 and C5

Other good Reading:

- A paper by H. Leff that compares a classical and Boson ideal gas, `PhotonGasAJP.pdf` is under the Resources label on the Week 10 page of our Moodle site.

Warmup Problems:

1: Counting states for three cases: We engineer a system where the single-particle energy levels are equally spaced. (Like for a harmonic oscillator.) Call the separation between adjacent energy levels Δ. The system is inhabited by 5 particles. Consider three cases:

1. They are identical fermions.

2. They are identical bosons.

3. They are distinguishable.

(a) For each case 1, 2 and 3... describe the state of lowest total energy, which we would call its "ground state".
(b) How does the ground state energy, E_{gs}, compare for the three cases?
(c) How many allowed states are there for case 1, 2 and 3 if the system has energy $E = E_{gs} + \Delta$?
(d) How many allowed states are there for case 1, 2 and 3 if the system has energy $E = E_{gs} + 2\Delta$?

2: Black hole Hawking radiation : Schroeder problem 7.53 part (a)

Problems to discuss in our meeting

Note: The * means that these problems are to be handed in. They are due the day after we meet.

1: Microcanonical derivation of the Bose-Einstein and Fermi-Dirac Distributions B&B Problem 29.6

2: The Cosmic Microwave Background B&B Problem 23.3

3: The Debye model
(i) G&T Problem 6.65 part (a)
(ii) G&T Problem 6.65 part (b) was originally assigned, but it doesn't make a lot of sense to me. It asks you to evaluate $E(T)$ numerically and then plot $C(T)$. This is tough if all you have to go on is $E(T)$. Please do the two items below instead?

- Use the formula found in G&T for the Debye solid's energy

$$E(T) = 9NkT(\frac{T}{T_D})^3 \int_0^{T_D/T} \frac{x^3}{e^x - 1} dx \qquad (1)$$

  to plot the energy $E(T)/9Nk$ vs. T at a set of values of T_D/T, that span a decent range. The challenge here is to evaluate the integral over x numerically for each value of its upper limit: T_D/T.

- Realize that you can find $C(T) = dE/dT$ by applying calculus skills to Eq. (1). And if you don't feel like doing it, B&B, Schroeder, and the website linked below have done it for us:

$$C(T) = 9Nk(\frac{T}{T_D})^3 \int_0^{T_D/T} \frac{x^4\ e^x}{(e^x - 1)^2} dx \qquad (2)$$

  Please do the needed integral in Eq. (2) and create a plot of $C(T)/9Nk$ vs. T.

In case it helps, here is a cool site with a lovely picture of waving phonons `https://eng.libretexts.org/Bookshelves/Materials_Science/Supplemental_Modules_(Materials_Science)/Electronic_Properties/Debye_Model_For_Specific_Heat`

4: Black body basics: G&T Problem 6.25

5*: Finding μ from N: Boson case: Schroeder problem 7.17

6*: Einstein and Debye Theories: G&T Problem 6.35

7: Energy and Entropy of a photon gas: B&B Problem 23.7

8: Black hole Hawking radiation (continued):
Schroeder 7.53 parts (b) - (e)