A helpful guide related to Week 9 Problem #4 and also some bonus material on Critical exponents:

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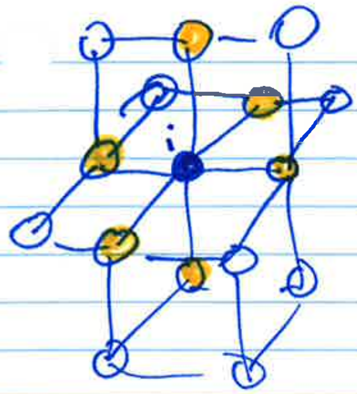


Mean field Theory

Each spin has q neighbors

(Schroeder uses "n" ... some other books use "z")

Spin i feels effective field

$$H_{eff} = J \sum_{j=1}^{q} s_j + H$$

Let's take mean effective field

$$\bar{H}_{eff} = J q \bar{s}_j + H$$

assume all s_j are same

$$= J q m + H$$

prettier notn... $\bar{s}_j = m$

Thence $Z_1 = \sum_{s_i = \pm 1} e^{+\beta s_i \bar{H}_{eff}}$;

$$Z_1 = 2\cosh[\beta(Jqm + H)]$$

$f = -kT \ln Z_1$;

$$f = -kT \ln(2\cosh[\beta(Jqm+H)])$$

Thus

$$-\frac{\partial f}{\partial H} = \boxed{\tanh[\beta(Jqm+H)] = m}$$

This must be solved self consistently... it is subject of **Problem 4** This week (G+T 5.18)

That problem even goes further... Lets us 'improve' on mean field as described in (G+T 5.17)

Improvement: $s_i = m + \Delta_i$; $s_j = m + \Delta_j$

smallish

$$\Rightarrow \quad E = -J \sum_{\langle ij \rangle} s_i s_j - H \sum_i s_i$$

$$= +J \sum_{\langle ij \rangle} m^2 - J\,m \sum_{\langle ij \rangle} (s_i + s_j) - H \sum_i s_i$$

To 1st order in Δ_i

$$E = + \frac{JqNm^2}{2} - \left(Jqm + H\right) \sum_{i=1}^{N} s_i$$

$$\Rightarrow \quad Z(T,V,N) = e^{-\beta \frac{JqNm^2}{2}} \left[2\cosh\left[\beta(Jqm+H)\right]\right]^N$$

$$\Rightarrow \quad f = \underbrace{\frac{1}{2} Jqm^2}_{\text{add'l term not on p. 1}} - kT \ln\left\{ 2 \cosh\left[\beta(Jqm+H)\right]\right]$$

improved

Critical temps ...

Critical Exponents

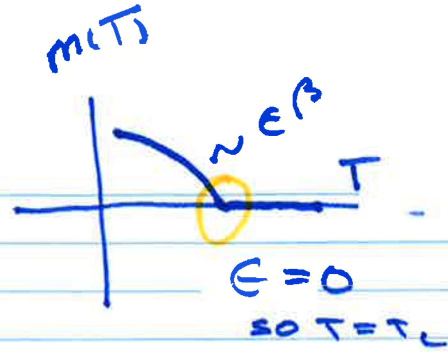
The Para → Ferro transition happens at a special T_c where spontaneous magnetiz'n occurs. Thus $m(H=0) \neq 0$ for $T < T_c$.

Also, as we let $T \to T_c$, there are power laws that important quantities obey.

eg.
$$m(T) = \begin{cases} 0 & T \geq T_c \\ \text{const.} \left(\frac{T_c - T}{T_c}\right)^\beta & T < T_c \end{cases}$$

$$\frac{T_c - T}{T_c} \equiv \epsilon \quad \text{a small number.}$$

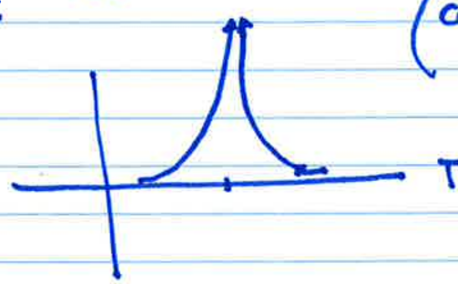
So



β is a <u>critical exponent</u>

These exponents are <u>same</u> ∀ lattices in a certain spatial dimension. Also, above a certain critical dimensionality, they are equal to their mean field values.

Also we'll see γ: $$\chi(\epsilon) = \begin{cases} \text{const.}\ \epsilon^{-\gamma} & \epsilon < 0 \ (T > T_c) \\ \text{const}'.\ \epsilon^{-\gamma} & \epsilon > 0 \ (T < T_c) \end{cases}$$



Finally, δ. This one involves being <u>at</u> T_c, and varying H near 0:

$$m(T=T_c, H) \sim H^{1/\delta}$$

i.e. $$H \underset{T=T_c}{\sim} m^\delta$$