

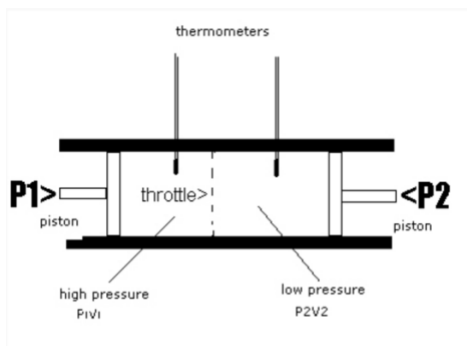
## About Week 3 Problem 9 The inversion temperature for a J-K process

1. Here is a great discussion of the work of Joule and Thomson (aka Lord Kelvin) that has narrative, explanations and pictures:

<https://carnotcycle.wordpress.com/tag/inversion-temperature/>

2. Some excerpts:

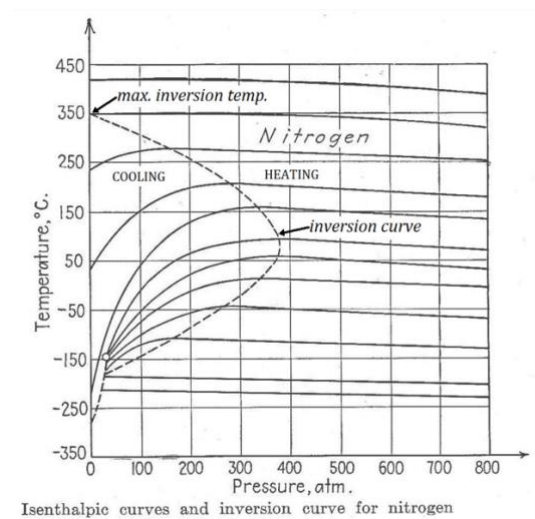
"Not surprisingly it led to discoveries, which turned out to have commercial as well as purely scientific value, since the Joule-Thomson effect made possible the liquefaction of gases and opened the way to important applications of throttling such as refrigeration."



Schematic



Practical use gas coolers, liquifiers and liquified gasses



Isenthalpic curves and inversion curve for nitrogen

The upper intersection of the inversion curve with the line of no pressure denotes the maximum inversion temperature for the given gas. Above this temperature the Joule-Thomson effect cannot produce cooling at any pressure.

Maximum inversion temperature of some gases

| Gas      | Maximum inversion temperature (K) |
|----------|-----------------------------------|
| Nitrogen | 620.93                            |
| Air      | 603.15                            |
| Hydrogen | 195.37                            |
| Helium   | 23.15                             |

The Joule-Thomson coefficient is defined:

$$\mu_{JT} = \left( \frac{\partial T}{\partial P} \right)_H$$

↑  
aka  $\mu_{JK}$

(People can't agree on whether to call the guy "Thomson" or "Lord Kelvin".)

Key idea: What is the sign of  $\mu_{JT}$ ? This coefficient,  $\mu_{JT}$ , is the slope of the "Isenthalpic curves" shown above. It depends on  $T, P$ ; and the curves are different for different gasses.

*Note: There is a maximum  $T$  above which you can't cool. That is the "maximum inversion temperature",  $T_{inv}$ , for the gas. **Below  $T_{inv}$ , in some decent range of temperatures and pressures, the gas can be cooled by a throttling (J-K) process.***

3. What did we find in Problem 9? At low densities,  $\rho$ ,  $T_{inv} = 2a/bR$ .

A larger  $a$  means a **stronger attraction** between molecules, and larger  $b$  means the molecules **repell more strongly** at close distances (that is their repulsive region is larger). Attractive and repulsive effects "balance" at  $T_{inv}$  and the gas behaves (to first order in  $\rho$ ) like an ideal gas, for which  $\mu_{JT} = 0$ .

4. Last thought: We might compare with the energy-conserving Joule expansion.

$$\mu_J \equiv \left( \frac{\partial T}{\partial V} \right)_U \quad ; \quad \mu_{JK} \equiv \left( \frac{\partial T}{\partial P} \right)_H$$

The coefficient  $\mu_J$  is always negative; expansion always leads to cooling. But  $\mu_J \sim a$  so the more strongly attractive the gas molecules are, the cooler the expanding gas becomes. Though these processes are different (Joule: energy conserving vs. Joule-Kelvin: enthalpy conserving) we can at least say that the stronger the attraction, the more "readily" the gas is willing to cool ... it has a higher  $T_{inv}$ .