Derivation of the AS Curve from the Labor Market Model when the PS line is flat [$m = m_0$]

Combining PS & WS:

$$(1) \quad P = P^e \frac{A^e}{A} (1+m_0)(1-\alpha U + z)$$

$$U = \frac{L-N}{L} = 1 - \frac{Y/A}{L}$$

where L is the size of the Labor Force

$$(2) \quad P = (1+m_0)\frac{A^e}{A} P^e \left[(1-\alpha+z) + \alpha \frac{Y/A}{L}\right]$$

$$\frac{P}{P^e} = \left[(1+m_0)\frac{A^e}{A}\right]\left[(1-\alpha+z) + \alpha \frac{Y/A}{L}\right]$$

By the natural rate hypothesis:

$$1 = \left[(1+m_0)\frac{A^e}{A}\right]\left[(1-\alpha+z) + \alpha \frac{Y_n/A}{L}\right]$$

$$\left(\frac{P}{P^e} - 1\right) = (1+m_0)\frac{A^e}{A}\frac{\alpha}{LA}\left[Y - Y_n\right]$$

$$(3) \quad P = P^e + (1+m_0)\frac{A^e}{A^2}\frac{\alpha}{L}P^e\left[Y - Y_n\right]$$

which we simplify to:

$$(4) \quad P = P^e + s_1\left[Y - Y_n\right]$$

where $S_1 = (1+m_0)\frac{A^e}{A^2}\frac{\alpha}{L}P^e$

Derivation of the AS Curve from the Labor Market Model when the PS curve is upward sloping $[m = m_0 - U]$

Combining PS and WS now yields:

$$(1)\ P = P^e \frac{A^e}{A}\left[1 + m_0 - U\right]\left[1 - \alpha U + z\right]$$

$$= P^e \frac{A^e}{A}\left[\begin{array}{l}(1+m_0)(1+z) - \{(1+m_0)\alpha + (1+z)\}U \\ + \alpha U^2\end{array}\right]$$

$$(2)\ \frac{P}{P^e} = \frac{A^e}{A}\left[\begin{array}{l}(1+m_0)(1+z) - \{(1+m_0)\alpha + (1+z)\}U \\ + \alpha U^2\end{array}\right]$$

There is nothing more we can do to simplify (2). If we substitute $U = 1 - \frac{Y/A}{L}$, it will only get uglier. But you may remember something from calculus called a Taylor Approximation. If you don't (and it is definitely not a requirement for this course), it states that if you want to approximate a function $f(x)$ at some value $x = a$, you do this with a series of derivatives. A first order (derivative) approximation looks like this:

$$f(x) \cong f(x)\big|_a + f'(x)\big|_a (x-a)$$

where $f(x)\big|_a$ and $f'(x)\big|_a$ mean evaluating $f(x)$ and its derivative at point a.

Applying this to equation (2) where we evaluate the approximation at U_n *

$$(3) \quad \frac{P}{pe} \cong f(U)\Big|_{U_n} + f'(U)\Big|_{U_n}(U - U_n)$$

where the $f(U)$ function is the right hand side of (2)/ P^e. The Natural Rate Hypothesis $\Rightarrow f(U)\big|_{U_n} = 1$, since at U_n, $P/p^e = 1$ and

$$f'(U)\Big|_{U_n} = \frac{A^e}{A}\left[2\alpha U_n - \{(1+m_0)\alpha + (1+z)\}\right]$$

$$(4) \quad \frac{P}{pe} \cong 1 + \frac{A^e}{A}\left[2\alpha U_n - \{(1+m_0)\alpha + (1+z)\}\right](U - U_n)$$

substituting $(Y_n - Y)/AL$ for $U - U_n$

$$(5) \quad P = pe + \underbrace{\left[(1+m_0)\alpha + (1+z) - 2\alpha U_n\right]\frac{A^e\, pe}{AL}}(Y - Y_n)$$

This monster expression is >0 and is the analog of s_1 in equation (4) of the simpler model on p. 1.