Economics 21 **Spring 2016**

Problem Set 6

Problem Set is due back Friday April 22. Some of the questions come from various editions of Blanchard Because different students have different editions, I have put a scan of the questions at the end of Problem Set. If you do not have access to the relevant American Edition of Blanchard, you should answer the questions on the scan.

1) Blanchard (5th Edition), p. 16, Dig Deeper Question 4:

a) Parts a), b) and c). For this question, you should start with 2008 when U.S. Output per Person was $43,800. For part c), you should answer the question by forming a ratio of Output per Person at 2.8% growth/Output per Person at 1.8% growth. This question does not use growth theory, just the miracle of compound interest (which is the opposite of discounting) - if you don't know what that is, ask me.

b) Use the Rule of 70 and amaze your friends with your ability to calculate compounding in your head. The rule is:

$$\text{Number of Years for a Variable to Double in Value} = \frac{70}{\text{Growth Rate of the Variable}}$$

where the Growth Rate of the Variable is 1.8 if the variable is growing at 1.8%. How long will it take U.S. Output per Person to double if the growth rate is 1.8%? If the growth rate is 2.8%?

2) Blanchard (6th Edition), p. 246, Dig Deeper Question 7: parts d), e), f) only. Note, this question assumes $g_n = g_a = 0$

3) Blanchard, (6th Edition), p. 246, Dig Deeper Question 9:

a) Parts b) and c) only. Use the production function from Question 7 (**NOT** the square root one).

b) There is a simple formula for the elasticity of the steady state level of Y/N with respect to changes in s (i.e. how much of a percentage change in Y/N results for a given percentage change in s).

$$\frac{\Delta(Y/N)\big/(Y/N)}{\Delta s\big/s} = \frac{\alpha}{1-\alpha} = \frac{1}{2} \text{ for } \alpha = 1/3$$

The formula is a calculus formula, so it holds exactly if you take derivatives (which we are not doing) but only approximately for discrete changes (like in question c where s increases from 15% to 20%). Still, if you use the midpoint of the discrete changes as the base values of s and Y/N, which are s = .175 and Y/N = 1.5215, it is pretty close. With these numbers, what does the formula give you for the elasticity?

c) We can use the Rule of 70 to figure out how long it will take for the economy to get halfway to its new steady state[1] once we know the rate at which it approaches the steady state. The general formula for that rate is:

Rate at which the Economy Approaches the Steady State = $(1-\alpha)(g_a + g_n + \delta)$.

$$\text{Number of Years to Close Half the Gap} = \frac{70}{(1-\alpha)(g_a + g_n + \delta)}$$

In using this formula, as in the Rule of 70, the growth and depreciation rates are whole numbers (not percentages), but α is a fraction.

In this problem $g_n = g_a = 0$. So, for this problem, how long will it take to get halfway to the new steady state?

4) Blanchard, (6th Edition), p. 264, Dig Deeper Question 6: part a) only. **Use the production function**:

$$Y = K^{\frac{1}{3}}(N \cdot A)^{\frac{2}{3}}$$

Given the numbers provided by Blanchard ($g_n = 2\%$, $g_a = 4\%$, and $\delta = 10\%$,), how long does it take for this economy to close ½ of the gap to the steady state?

5) For the actual U.S. economy since 1870, $g_n = 1.85\%$, $g_a = 1.15\%$, and $\delta \approx 8\%$, how long does it take for the economy to close ½ of the gap to the steady state?

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[1] Half-lives also follow the Rule of 70. So we can ask "how much of a gap between the old value of Y/N and the new value of Y/N did the increase in s create"? That gap will ultimately be closed and go to zero, and the Rule of 70 tells us how long it will take to close half of the gap.

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4. The New Economy and growth

The average annual growth rate of output per worker in the United States rose from 1.8% during the period 1970 to 1995 to 2.8% for the years 1996 to 2006. This has led to talk of a New Economy and of sustained higher growth in the future than in the past.

a. Suppose output per worker grows at 1.8% per year. What will output per worker be—relative to today's level—in 10 years? 20 years? 50 years?

b. Suppose output per worker grows instead at 2.8% per year. What will output per worker be—relative to today's level—in 10 years? 20 years? 50 years?

c. If the United States has really entered a New Economy, and the average annual growth rate of output per worker has increased from 1.8% per year to 2.8%, how much higher will the U.S. standard of living be in 10 years, 20 years, and 50 years relative to what it would have been had the United States remained in the Old Economy?

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p. 246 *7. The Cobb-Douglas production function and the steady state. This problem is based on the material in the chapter appendix. Suppose that the economy's production function is given by*

$$Y = K^{\alpha}N^{1-\alpha}$$

and assume that $\alpha = 1/3$.

a. Is this production function characterized by constant returns to scale? Explain.

b. Are there decreasing returns to capital?

c. Are there decreasing returns to labor?

d. Transform the production function into a relation between output per worker and capital per worker.

e. For a given saving rate, s, and depreciation rate, δ, give an expression for capital per worker in the steady state.

f. Give an expression for output per worker in the steady state.

g. Solve for the steady-state level of output per worker when $s = 0.32$ and $\delta = 0.08$.

h. Suppose that the depreciation rate remains constant at $\delta = 0.08$, while the saving rate is reduced by half, to $s = 0.16$. What is the new steady-state output per worker?

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9. Deficits and the capital stock

For the production function, $Y = \sqrt{K}\sqrt{N}$, equation (11.8) gives the solution for the steady-state capital stock per worker.

a. Retrace the steps in the text that derive equation (11.8).

b. Suppose that the saving rate, s, is initially 15% per year, and the depreciation rate, δ, is 7.5%. What is the steady-state capital stock per worker? What is steady-state output per worker?

c. Suppose that there is a government deficit of 5% of GDP and that the government eliminates this deficit. Assume that private saving is unchanged so that national saving increases to 20%. What is the new steady-state capital stock per worker? What is the new steady-state output per worker? How does this compare to your answer to part (b)?

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p264 *6. Suppose that the economy's production function is*

$$Y = \sqrt{K}\sqrt{AN}$$

that the saving rate, s, is equal to 16%, and that the rate of depreciation, δ, is equal to 10%. Suppose further that the number of workers grows at 2% per year and that the rate of technological progress is 4% per year.

a. Find the steady-state values of the variables listed in (i) through (v).

   i. The capital stock per effective worker.

   ii. Output per effective worker.

   iii. The growth rate of output per effective worker.

   iv. The growth rate of output per worker.

   v. The growth rate of output.