AFFECT AND MATHEMATICAL PROBLEM SOLVING
A New Perspective

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Affect and Learning: Causes and Consequences of Emotional Interactions

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Affect is the least investigated aspect of human problem solving, yet it is probably the aspect most often mentioned as deserving further investigation. The "problem-solving" and "teaching-and-learning" literature is full of remarks that have a single message: "Someday soon—maybe tomorrow—we must get around to doing something about affect and emotion." I am delighted to see that "tomorrow" has come.

In a sense, however, tomorrow came some time ago, and I have been puzzled by the fact that an extensive literature on stress and cognitive efficiency rarely, if ever, crossed the disciplinary fences and reservations. I refer here to the continuing interest in the way motives, stress, emotions, and affects interact with the acquisition of skills, and the storage and recovery of memories. That particular tradition goes back to the beginning of the century with the Yerkes-Dodson law, which introduced the notion that the relation between stress and efficiency of learning was curvilinear—worst at both weak and strong levels of stress and best at intermediate levels (Yerkes & Dodson, 1908). In modern times, it is represented by Easterbrook's influential paper (1959), in which he hypothesized that our attentional capacity for picking up information is increasingly impaired by increases in emotional intensity. Major treatments within the information-processing context were provided by Broadbent's *Decision and Stress* (1971) and Kahneman's *Attention and Effort* (1973). I have discussed these various approaches in the context of my own view of stress and efficiency in my book *Mind and Body* (1984), and I develop this theme further within appropriate contexts to follow.

First, I present a brief outline of my constructivist view of emotion.[1] This outline is followed by a discussion of some possible applications of those notions to problem solving and learning. A microanalytic approach is stressed, and questions are asked about the uses of affect and the specific effect of human error.

Before I proceed, a brief word about the word *affect*. Unfortunately, the term has meant many things to many people, acquiring interpretations that range from "hot" to "cold." At the hot end, affect is used coextensively with the word *emotion*, implying an intensity dimension; at the cold end, it is often used without passion, referring to preferences, likes and dislikes, and choices. I interpret the use that it has acquired in the problem-solving field to be hot, rather than cold, and I frequently use the words *affect* and *emotion* interchangeably in this chapter.

The Construction of Emotion

Much contemporary cognitive theory would lead one to believe that human beings are unable to feel. Until recently, conventional wisdom in cognitive science described people as passionless creatures who think and act rationally and coolly. Such an approach has been most obvious in the investigation of problem solving. In reality, psychologists and other human beings typically are frustrated, angry, joyous, delighted, intense, anxious, elated, and even fearful when dealing with complex problems, such as a recalcitrant computer program, an exasperated customer, a difficult bridge hand, an unassembled toy, or a mathematics problem. If cognitive psychology aspires to an understanding of human thought and action, it can ill afford to leave out their emotional aspects. I should add that it is not enough for cognitive models to have nodes or processors labeled *fear* or *joy* that can be accessed whenever appropriate. That approach simply acknowledges theoretical impoverishment; it does not solve the problem of how emotions arise or how they are to be represented.

Emotion is not only anecdotally and phenomenally part of human thought and action; there is now a burgeoning body of evidence that emotional states interact in important ways with traditional cognitive functions. For example, Isen's work has shown that positive feelings determine the accessibility of mental contents in the process of decision making, serve as retrieval cues, and influence problem-solving strategies (e.g., Isen, Means, Patrick, & Nowicki, 1982). More generally, accessibility of mental contents is determined by the mood both at the time of original encounter and at the time of retrieval (Bower, 1981; Bower & Mayer, 1985). Thus, emotion should be of both general and specific interest to cognitive psychologists.

There are essentially two different views of emotional phenomena. One view is that emotions are discrete patterns of behavior, experience, and neural activity. These patterned packages are considered to be fundamental emotions (such as fear, joy, and anger), and are usually restricted to some 5 or 10 in number. I shall refer to this as the fundamentalist position, with Charles Darwin as its patron saint. It is somewhat surprising that after more than 100 years of searching, the fundamental emotion theorists cannot agree on what these *basic* emotions are. The basic emotions usually are considered to have developed as a function of human (and mammalian) evolution, with other emotions being some combination of the fundamental ones. Given the wide variety of human emotion, this latter postulate engenders complex analyses of and recipes for emotion—something like a cordon bleu school of emotion.

The second approach is cognitive and constructivist, which considers emotional experience (and behavior) to be the result of cognitive analyses and physiological (autonomic nervous system) response (Averill, 1980; Lazarus, Kanner, & Folkman, 1980; Mandler, 1984). The founding father of the constructivist approach is William James, but his particular constructions depended entirely on patterns of visceral and muscular feedback and are no longer considered acceptable.

Another dichotomous argument in the field of emotion concerns the character of emotional or affective reactions. This argument is related to, but somewhat different from, the division between fundamentalist and constructivist theories. The distinction here is between a constructivist view of emotional reactions (specifically subjective feelings) and one that makes a distinction between affective and cognitive analyses. The foremost practitioner of the latter approach is Robert Zajonc. He has marshalled an impressive array of anecdotal and phenomenal evidence to argue that affective responses are unmediated and are fast initial reactions to people and events (Zajonc, 1980). Affective reactions are said to occur (without mediating cognitive analyses) in response to specific aspects of an event. The implication is that specific characteristics of events (so-called "preferanda") force preferences. The response of a cognitive constructivist to this view is that the attributes and features in the environment need to be analyzed and processed by underlying representations. Specifically, the phenomenal evidence that many of our reactions to the world are initially affective is not, by itself, evidence for an unmediated affective response. Affective experiences are constructed conscious contents, just as any other such contents. The indisputable observation that we frequently react affectively to events, before experiencing a more "analytic" knowledge of the event, speaks to the ubiquity of affective and evaluational constructions and intentions. We live in a world of value and affect, and the themes that determine our conscious constructions often require an affective content. This does not force an absence of other analyses and activations going on at the same time at the preconscious level. The analyses to be used in conscious constructions will be determined by the intentions and requirements of the moment, which, in many cases, happen to be affective. On the other hand, the assertion of "fast" affective reactions is an empirical one. If, in fact, affective, evaluative reactions are faster than those that require only access to cognitions (knowledge), then one would have to reconsider the constructivist argument. However, the available evidence suggests that affective reactions are actually *slower* than cognitive ones (e.g., Mandler & Shebo, 1983). Such a finding is consistent with the notion that questions about the familiarity or identity of objects or events require access to relatively fewer features of the underlying representation and can be quickly constructed; by comparison, questions about the appeal, beauty, or desirability of the object or event require more extensive analyses and constructions.

The construction of emotion, in general—at least as seen by this practitioner—consists of the concatenation in consciousness of some cognitive evaluative schema with the perception of visceral arousal. We experience unitary emotions, often labeled with varying consistency with such names as *fear, anger, anxiety,* and *jealousy.* Only rarely do we experience more than one emotion at one time. These conscious constructions are, as are all other such constructions, holistic unitary experiences, even though they may derive from separate and even independent schematic representations. This view of the construction of conscious experience suggests that the conscious contents of the moment are constructed out of two or more underlying structures and have the function of "making sense" of the current situation (Mandler & Nakamura, 1987). This attitude toward emotion only approximates the common-sense meaning of the term. The question "What is an emotion?" is not, in principle, answerable. The term is a natural language expression that has all the advantages (communicative and inclusive) and disadvantages (imprecise and vague) of the common language. It is precisely for communicative purposes, however, that one needs to approximate the common meaning as a first step.

Of the possible analyses of common language emotions, I have focused on two characteristics: the notion that emotions express some aspect of value, and the assertion that emotions are hot, implying a gut reaction or a visceral response. These two aspects not only speak to the common usage, but they also reflect the fact that conscious constructions are frequent occurrences in everyday life; thus, an analysis of the concatenation of value and visceral arousal addresses both natural language usage and a theoretically important problem. One of the consequences of such a position is that it leads to the postulation of a potentially innumerable number of different emotional states, because no situational evaluation will be the same from one occasion to the next. There are, of course, regularities in human thought and action that produce general categories of these constructions–categories that have family resemblances and overlap in the features that are selected for analysis and that create the representation of value (whether it is the simple dichotomy of good and bad, the appreciation of beauty, or the perception of evil). These families of occasions and meanings construct the categories of emotions found in the natural language (and psychology). The commonalities found within these categories may vary from case to case, and they have different bases for their occurrence. These commonalities may be based on the similarity of external conditions, as in the case of some fears and environmental threats. Sometimes an emotional category may be based on a collection of similar behaviors, such as the subjective feelings of fear related to flight. The common category may arise from a class of incipient behaviors, as in hostility and destructive action. Hormonal and physiological reactions may provide a common basis for the experience of lust. Purely cognitive evaluations may generate judgments of helplessness that eventuate in anxiety. These commonalities give rise to the appearance of fundamental or discrete emotions. With respect to the neural aspect of discrete emotions, I do not believe that we have heard the last word on the assertion that certain emotions are identified with specific localized brain functions. The function of the limbic system, for example, in the generation of emotional experience is vague, but the elicitation of specific actions (e.g., "rage" in cats) can be seen as just that–actions. We do not know whether such elicited actions produce any accompanying subjective emotional experiences. I would argue initially that such actions may be constituents in the construction of subjective emotions.

Emotions are usually situation specific, and subjective emotional states, however one defines their source, need to be tied to cognitive evaluations that "select" the appropriate emotion. The various indices that constitute an emotion are not haphazard collections of current conditions; they are organized by behavioral, cognitive, or physiological states and conditions of the individual. The source for the discreteness of the emotions can often be found in those conditions. One should understand, however, that even these "fundamental" emotional states require some analytic, cognitive processing. Even if it were the case that well-coordinated neural/behavioral/cognitive systems are "elicited" as a unit, as single emotions, one would still have to analyze the eliciting conditions that produce one emotion or another.

The problem of cognitive evaluation seems to be common to all emotion theories. In recent years, there has been an active search for the basis of these valuative structures. Basically, cognitive evaluations require a theory for the representation of value. That task cannot be avoided if we are truly interested in the full range of human thought and action. What is the mental representation that gives rise to judgments and feelings of good or bad or of some affective nature in general? Surprisingly, psychologists have paid relatively little attention to problems of value, in the sense of developing a theory of the underlying structures that give rise to phenomenal experiences of value. My concern is with both accepting and explaining these phenomenal experiences.

A full discussion of the problem of value is precluded in this presentation. In brief it involves the different external and internal sources that lead us to see some person or event as good or bad, as evil or benign, as harmful or beneficial. Within the realm of learning and problem solving, there is a wide range of sources of values. On the one hand, it is considered a good thing to be proficient in mathematics; on the other hand, mathematics is a source of apprehensions. The well-known phenomenon of *number shock* demonstrates an underlying value that mathematical manipulation is difficult, complex, and potentially frightening. Many people in our society confess helplessness when dealing with numbers; others admire those who perform even arithmetic without pain. Also relevant to the problem-solving context is the value inherent in the completion of a problem or a task. The very fact that a task is done, often regardless of whether it is completed successfully, provides a source of positive value. All of these values will feed into the construction of emotions during mathematical problem solving. More specific is the subjective evaluation of an error, a mistake, or a success in the process of cognitive processing,[2] which will be addressed in more detail later in this chapter.

If evaluative cognitions provide the quality of an emotional experience, then visceral activity provides its intensity and peculiar emotional feel. Some theorists, and some critics of my position, have argued that the autonomic, visceral component is not necessary for emotional experience. In reply, I would argue that emotion is, of course, one of those great natural language concepts that can serve any and most arguments, depending on how one wishes to use the language or the concept. The best we can do is to propose a definition that satisfies a reasonable portion of the common concept and produces some degree of social consensus. In part, the denial of visceral activity as a necessary part of "real" emotions is related to such alternative terms as *affect*. One may say that something is pretty or fine or awful or even disgusting quite dispassionately and unemotionally. I believe that we need to understand the occasions when visceral activity, however slight, co-occurs with these judgments or affects.

In one version of the common understanding of emotion, the occurrence of some visceral or gut reaction is generally assumed. Emotions are said to occur when we feel aroused, agitated, when our "guts are in an uproar," and so forth. Reference is always made (and properly so) to some autonomic nervous system activity, such as increased heart rate, sweating, or gastrointestinal upheavals. The autonomic nervous system (ANS) has been implicated in quasi-emotional activity ever since Walter Cannon delineated the function of the sympathetic and parasympathetic systems in fight/flight reactions, giving them a role beyond that of expending and conserving energy in keeping the internal environment stable. If one looks at the literature on the ANS, however, one is faced with a lack of any principled account of the sources of ANS activation.

I have argued that on a majority of occasions, visceral arousal follows the occurrence of some perceptual or cognitive discrepancy or the interruption or blocking of some ongoing action. Such discrepancies and interruptions depend to a large extent on the organization of mental representation of thought and action. Within the purview of schema theory, these discrepancies occur when the expectations of some schema are violated. This is the case whether the violating event is worse or better than expected and accounts for visceral arousal in both unhappy and joyful occasions. Most emotions follow such discrepancies because the discrepancy produces visceral arousal. The combination of that arousal with an ongoing evaluative cognition produces the subjective experience of an emotion. I do *not* say that emotions are interruptions. Interruptions, discrepancies, blocks, frustrations, novelties, and so forth, are occasions for ANS activity.

Discrepancies may occur for a variety of different expectations. We are rarely operating under the guidance of a single schema. Schemas are arranged hierarchically, and a number of them are active at the same time; thus, when working on a mathematical problem, one expects, for example, that the pencil one is using will not break, that one can remember the meaning of symbols, and that the functions of symbols within a particular equation will be the usual ones. In addition, certain parallel expectations (schemas) lead one to expect that the electricity will not fail, that one's chair will not collapse, that, in short, the world is more or less stable.

Whether or not an emotional construction follows the arousal produced by a discrepancy depends on the evaluative activity of the individual. It is the concatenation of an evaluative process and ANS arousal that produces emotion. Note that I am not talking about conscious appraisals of the situation when I call on evaluative processes. The appraisals may be conscious, but they typically appear in consciousness only as components of the holistic emotional experience.

We now have some rather convincing evidence that any sort of discrepancy produces autonomic arousal. One study performed by Yoshi Nakamura in our laboratory presented subjects with very simple, boring stories and asked them to imagine how the stories would end. The stories were presented one sentence at a time, and the last sentence was either consistent or discrepant with the preceding context. In all cases, we obtained slight, but significant, increases in heart rate variability following discrepancy. In a study of an interactive computer game, we got large increases in heart rate on the order of 10 to 25 beats per minute in response to unexpected events.

The effects of situational or life stress are excellent examples of unexpected events that produce visceral arousal, negative or positive evaluations, and emotional experiences. Berscheid (1983) has imaginatively described the conditions of interpersonal interactions that lead to interruptions and discrepancies and, therefore, to emotional reactions. When a relationship is meshed (when the actions of one individual depend on the actions of the other), then the two people involved may become occasions for each other's interruptions. The actions of the other are essential for one's own action; thus, emotional reactions are more likely in such meshed relationships than they are in "parallel" relationships, in which the actions of two individuals are not interdependent. A similar analysis can be applied to the relationship between task and learner. When the two are meshed, when any step by the learner implies a particular serial dependency in the task, then interruptions will have severe consequences. In a more parallel situation, however, the learner will have available diversions and alternatives that are not entirely constrained by the task.

Finally, the construction of emotions requires conscious capacity. The experience of emotion is, by definition, a conscious state and, thus, pre-empts limited capacity. Limited capacity refers to the fact that conscious contents are highly restricted at any one point in time. Whenever some particular construction pre-empts conscious capacity, then other processes that require such capacity will be impaired. The best example of this is found in stress and panic reactions, when emotional reactions prevent adequate problem-solving activities. As a result, emotional experiences are frequently not conducive to the full utilization of the cognitive apparatus; thought may become simplified (i.e., stereotyped and canalized) and tend to revert to simpler modes of problem solving. The effects are not necessarily intrusive and deleterious, however. In part, it will depend on other mental contents that are activated by the emotional experience and that may become available for dealing with situations. The relationship of emotions to discrepancies and ANS recruitment also points to their adaptive function; emotions occur at important times in the life of the organism and may serve to prepare it for more effective thought and action when focused attention is needed.

I have discussed the interactions of stress and cognitive efficiency at length in *Mind and Body* (1984). A survey of the literature supports the conclusion that stress tends to decrease attention to peripheral events and to focus attentional conscious capacity on those aspects of the situation that the individual considers important. But there lies the rub. What a particular individual considers to be important may not lead to a solution of the current problem; thus, stress can be helpful in screening out irrelevancies, but it may also result in focusing on the wrong strategy. In order to handle stress and affect effectively in a problem-solving task, the individual must be equipped with adequate knowledge of the task and possible ways of solving it. In other words, inadequate information leads to stress, but the well-informed individual can use stress constructively.

It has been argued that discrepancy is vacuous as an explanatory argument because discrepancy, as usually defined, is practically always present. To some degree, all events are somewhat discrepant from what is expected; the world changes continuously. This is correct, of course, and I would expect that some degree of arousal is present in many, possibly most, day-to-day situations, but so is some degree of feelings or moods. In fact, the criticism has been made that theories such as the cognitive/arousal theory are too discrete, that they do not account for the pervasiveness of moods and emotions, and that human beings are characterized by some feeling or mood state much of the time. The pervasiveness of discrepancies, which clearly cannot be denied, accounts for the continuous feeling states; however, the degree of discrepancy is usually slight, and the amount of arousal is small, which accounts for background feelings and moods. So-called "true" emotions typically occur with high degrees of arousal and are frequently associated with extreme discrepancies and interruptions.

I need to dwell a little on the positive emotions – the joys and pleasures of learning and problem solving, for example. Many otherwise sympathetic consumers of discrepancy theory feel uncomfortable with the notion that discrepancies are the source of autonomic arousal for positive as well as for negative events. One of the reasons for this discomfort is the common notion that discrepancy is somehow itself a negative event and is associated with frustration and other similar concepts. It is not, of course, within the confines of this theory. Consider the joys of young love. One has met the person of one's dreams and hopes, but reciprocity is not quite apparent. One is to meet again a few days hence; as the object of passion appears at the designated time, joy floods the lover, and ecstasy is near. What is discrepant? I would argue that the anticipation of the event is never devoid of doubts and fears (Will the loved one appear at the appointed time? Is he or she at all interested? Does he or she look as desirable as one has imagined?). The world of romantic love is full of such ambivalences, and whenever there are ambivalences, the actual event will be discrepant with some of them. There is no argument about the emotional quality, the "value," of the love; what is at stake is whether there is accompanying autonomic arousal generated by interruptions and discrepancies. In contrast, consider a positive event that is fully anticipated, in all its details and nuances. A prize has been won, and the check arrives in your mail. The value is still there, but the intensity will be relatively low. I believe that an appropriate analysis of positive events will disclose the operation of many ambivalent expectancies that are more than sufficient to explain the intensity of positive emotions. Of course, similar ambivalences operate for many negative

events. For every expected good, there are thoughts of disappointments and slip-ups; for every expected bad, there are hopes of redemption and relief.

Similar arguments apply to the pleasures of learning, and to mathematics, in particular. For example, when a youngster first encounters mathematics, it may be strange and unusual, itself a source of arousal. But the mastery of the subject, the mastery of the novel, shows it to be not as strange and possibly frightening as it first appeared; the discrepancy is positively resolved, and positive emotions emerge. Finding solutions to problems that at first seem forbidding shows a similar concatenation of events. New ideas are usually discovered in the context of uncertainties; their effect can, as we all know, be quite exhilarating – for some of us, as exhilarating as the joys of young love.

This brief review should convey the flavor of my theoretical position. My major interest here is to focus on the learning process as it generates discrepancies and interruptions – mainly in the production of errors and unexpected successes, as well as in the values (the evaluative reactions) that may arise in the course of the learning process.

Before I describe an application of discrepancy theory to problem solving, I wish to discuss an older tradition in the affect/learning domain – the exploration of the covariation of individual differences and task performance.

Macroanalysis: Individual Differences and Cognitive Efficiency

One traditional approach to the relation between emotion and cognitive functioning has been to look at the way in which variation in some affective characteristic of individuals covaries with one or the other measure of their cognitive functions. I call this the *macroanalytic approach* because it typically generates global measures of both individual variation and task performance. The central interest is in the individual's performance on a task or test as a whole, which is often expressed in a single score on the task. There is little room in such an approach for a distinctly different goal – that is, how to design tasks for people so that performance can be optimized, or at least minimally affected by people's nonadaptive preoccupations and predilections (such as test anxiety). The general loss of appeal of the macroanalytic approach in recent years was, of course, influenced in large part by the rather small effects of the covariations between affect and cognitive performance (i.e., correlations of about .30, accounting for less than 10% of the variance on the problem tasks).

I first became involved with the macroanalytic approach 35 years ago, when Seymour Sarason and I studied the relation between scores on a test-anxiety scale and various memorial and problem-solving tasks (Mandler & Sarason, 1952).[3] I wrote the epilogue on my participation in that effort in 1972, pointing out the often atheoretical search for correlations that had typified the field and, more important from my point of view, the need to step back and look at (or better, *for*) theory as our primary concern. I called for more investigation into the actual operation of the processes that generate helplessness and disorganization (Mandler, 1972). In that same volume, I also suggested that test-anxiety scales may well be renamed *test-relevant-self-instruction scales* to document the fact that individuals' anxiety arises out of their specific interpretation and use of events as they unfold in the course of a test. Microanalyses of affect and cognition were needed – that is, what happens specifically in the interaction between the individual and the problem-solving task.

I want to raise one issue about the problem of individual differences in affect as it applies to complex thinking. One macroanalytic approach is to develop a test to measure individual variation and then to determine how performance on a variety of tasks covaries with that measure across a group of typical, or not-so-typical, individuals. Another approach, which is used very infrequently, is to assemble a population of representative tasks (rather than individuals) and to determine the extent to which each of the tasks produces affective reactions on a group of individuals. That produces a ranking of tasks rather than individuals, which is a quite different metric. In the best of designs, one would want to use both approaches, thus ranking both people and tasks. I believe we need to think more analytically about both kinds of approaches because a correlation of characteristics across individuals and a correlation of the *same* characteristics across tasks may produce very different results (e.g., Mandler 1959); for example, we can ascertain degree of test anxiety for a group of individuals, or we can develop an array of tasks that differ in the degree to which they elicit test anxiety. The correlation between test anxiety and effective problem-solving strategies may be negative for individuals, but nonexistent or even positive for the tasks. Which of these correlations turns out to be the one that we find important in designing curricula or teaching methods will depend on the problem that we wish to solve and the theory that informs our approach. We may be interested in designing an array of tasks; we may be interested in the melioration of individuals' difficulties with the tasks; or, optimally, we may be interested in some combination of the two. We cannot assume, however, that a particular correlation (e.g., for individuals) is an absolute measure of covariation of the two characteristics.

Microanalysis: Discrepancy Theory and Human Error

Any discrepancy in the course of problem solving represents a potential affective episode. As I noted earlier, such episodes must be seen within the context of a general flow of affective and moodlike changes. However, it is possible to identify specific kinds of discrepancies and interruptions that may occur in the course of problem solving, in general, and in mathematical reasoning and learning, in particular. The major class of such events can generally be termed *errors*, when the learner does something or thinks something that is different from his or her original intention or is different from what ought to happen. I shall discuss errors as such shortly, but we must keep in mind that people frequently engage in actions that they believe to be correct (i.e., they proceed as intended, but the actions are in fact false, incorrect, illogical, etc.). In the case of unintended errors, the discrepancy arises because of a mismatch between what is intended and what occurs; in the other cases, the mismatch is between an expected outcome ("I thought I did what would solve the problem") and the real-world response ("It didn't work"). Most intended and unintended errors are coupled with a negative evaluation of the current situation. These errors will usually result in negative affect (unhappiness, disgust, despair) in varying degrees of intensity. The result is interference with ongoing cognitive processes because of both the pre-emption of conscious capacity and the search for correction (looking for the source of the disruption). If this process is allowed to go unchecked (i.e., if the learner is continuously producing errors), the intensity of negative affect will increase. Such a sequence may eventually produce: abandonment of the task because it is too noxious to tolerate, panicky quasi-random attempts at solution, and general disorganization.

The other general class of discrepancies may be termed *successes*, which also may be intended or unintended. The intended success is a step in the solution process that works; the unintended success occurs when a particular action or thought produces unexpected positive results ("I just tried that because I couldn't think of anything else, and it worked"). In general, it is likely that the smooth progression of some planned course of problem solving will produce little in the way of arousal. After all, if I go through some cookbook steps toward solution, the expectation is that each step will be successful, and no discrepancy occurs; however, when I am not sure of the successive steps, success may be slightly arousing. During the early stages of the learning process, when the learner is unsure, we expect – and, of course, find – more affective states and more interference with learning. The well-practiced solution runs off smoothly and, by definition, produces little affect. From a more global perspective, the final solution of a problem will produce discrepancy if the learner is unsure of his or her ability; joy, delight, and satisfaction result.

Norman (1981) analyzed action slips within the general context of schema theory. Well-learned action sequences (and by implication, thought sequences) are specified by a high-level schema (the intention), which then recruits lower level schemas that guide the various components of the developing sequence. These component schemas are presumably activated both by the original intention (and its descendants) and by appropriate conditions of the task as the sequence (the problem solving) develops – that is, the thought sequence emerges interactively from both top-down and bottom-up sources. Norman discusses three categories of slips: errors in the formation of an intention, defective activation of component schemas, and defective triggering. In another paper, Norman (1980) makes the distinction between mistakes, which result from errors in the formation of an intention, and slips, which are errors in the execution of an intention. I cannot discuss these papers in detail, but they are worthy of attention for the purpose of analyzing specific tasks and the kinds of mistakes and errors that can occur.[4]

Errors: What Are They Good For?

The typical work on human errors and slips pays hardly any attention to problems of the consequences of the errors and essentially no attention to any affective consequences. Norman (1980), for example, is much concerned with problems of limited conscious capacity (he refers to it as *short-term-memory* capacity), but the possibility that affective consequences of errors demand and pre-empt such capacity is not seriously entertained. Because I am not concerned here with the causes or categorization of errors, but rather with their affective consequences, I want to emphasize exactly that point. The affect that occurs when errors or mistakes are made is a conscious event; it pre-empts our limited conscious capacity and has several consequences. First of all, just the perception of autonomic arousal, when severe, is demanding enough to hinder one's ability to act effectively ("I am so upset, I can't think straight"). Second, the occurrence of strong negative affect produces an immediate attempt to remove the reasons (the cause). That may be a useful consequence when the removal of the offending event is more important than a continuation of the problem-solving activity. In the usual learning context, however, the latter may be more adaptive. Third, even a positive affective experience in the context of problem solving may be deleterious. The individual solves some subroutine and stops, in part to admire the achievement, in part to savor the affect. In the process, the current contents of consciousness are displaced; one loses one's place in the stream of problem solving.

Is it the case, then, that errors and mistakes should be avoided whenever possible, and that tasks should be designed so that problem solving can be learned without errors? I now come to the subject of errorless learning: Can it happen? Is it a good thing? I shall return to the real world soon and discuss the inevitability of errors and mistakes and the advantages of having had prior experiences with them.

The concern with errorless learning started about 25 years ago when Skinner (1961) argued that learning should involve as few errors as possible, and that it should be possible (e.g., by making incremental steps small enough) to acquire skills and knowledge without the nuisance and debilitating effects of errors. Skinner provided little, if any, empirical evidence for his argument, but Terrace (1963, 1972) demonstrated an analogue of human errorless learning in animal discrimination training. The animal is required to distinguish between a positive stimulus and a negative stimulus. The required response must be made when a positive stimulus is presented and omitted when a negative stimulus is presented. Terrace showed that by slowly bringing in the negative stimulus, the animal (typically the pigeon) could learn to discriminate between a positive stimulus and the negative stimulus without making many (or any) responses to the negative stimulus. The major claim about errorless learning that concerns me here is that one of the by-products of discrimination learning does not appear following errorless learning. That by-product is emotional (and aggressive) behavior when a negative stimulus is presented.[5] It has been known for quite some time that negative stimuli (and the absence of reinforcement during extinction) produce emotional behavior in animals. A major theoretical position in another branch of behaviorist theory—Amsel's frustration theory—addresses these effects (e.g., Amsel, 1962).[6] If a method could be found that eliminated the emotional consequences of negative stimuli (and their avoidance), it would have important consequences for theories of learning and teaching; however, that is not the case. Rilling and his students (see Rilling, 1977, for a review) showed that most of the by-products of discrimination learning, and in particular aggressive emotional behavior, also occur following errorless learning. These behaviors "bear little relationship to the occurrence or nonoccurrence of errors during S- [negative stimulus]" (Rilling, 1977, p. 467). In keeping with the expectancy notions of interruption theory, "withdrawal of opportunity for reinforcement is one of the primary determinants of extinction-induced aggression" (Rilling, 1977, p. 469). It appears that it is the absence of positive stimuli (the expectations developed during the errorless responses to the positive stimulus) that produces arousal and emotional behavior.[7]

I am not advocating that one look to the animal learning literature for an understanding of human cognitive processing; however, the only data available suggest that errorless learning does not produce an affect-free learning environment. If we assume that there are, at present, no methods that produce painless learning and that the consequences of learning, no matter how painless, may still produce affective reactions, how do we accommodate such a state of affairs?

My main argument should be obvious by now. I do not believe that it is possible for an individual to live a cocooned life in which no failures, no mistakes, no slips will be encountered. These missteps in life (including mathematical learning) can be highly deleterious if totally unexpected; they are less interfering and less intense if their occurrence is a normal feature of life and, therefore, expected to occur at some time or another. The occurrence of affective reactions to learning experiences, arousal due to discrepancies during the learning process, and other bad experiences build up expectancies, and are built up into experiential schemas. Events that are totally unexpected produce maximal arousal and affective reactions. If, however, we know that errors (and very specific errors) can occur in problem-solving situations, the errors are not surprising and their consequences are not unexpected. The cocooned individual has extreme reactions to novel situations. In short, I believe that it is advantageous—at least in the problem-solving situation—for an individual to have been exposed to the "school of hard knocks." There is a nice set of demonstrations available in the animal (and human) experimental literature on the partial reinforcement effect. If a response is consistently reinforced during learning, then extinction of that response once reinforcement is absent will be relatively rapid; however, if the initial reinforcement is only given on some percentage of the acquisition trials, then extinction of the response will be slow. It is the experience with unreinforced responses that makes them less effective during extinction.[8]

How, then, do we design learning and teaching tasks? It is here that the notion of microanalysis receives its most direct application. We need to analyze the task and the learner to ensure that the surprises, errors, and missteps during acquisition are relatively minor—that is, that they can be mastered by some alternative route, by substitute actions, or by a restatement of the problem (one's intentions). At the same time, we do not want to avoid such affective incidents altogether; we want the learner to be prepared for them in the future. Thus, small incremental steps, specific instruction in subroutines, attention to possible difficulties, and instructions for the anticipation of difficulties play a role in analyzing and constructing a task. We also need to take into account the individual's attitudes and beliefs about the problem, because they will interact with the expectations that will be developed and perhaps be confirmed or violated.

Do We Want Affectless Learning?

The major message of the foregoing section is that affectless learning is not a possible goal for a theory or for the praxis of instruction. Common sense tells us that emotions and affective reactions are with us now and forever. Not surprisingly, the usual measures of individual differences tell us the same thing. The macroanalytic methods, no matter how unsatisfactory, do give us some indication of the person's affective preoccupations with tests, cognitive tasks, and similar situations. These dispositions will interact with the emotional consequences of specific errors, failures, successes, and strategies; thus, one would expect an individual with a high level of anxiety or arousal to produce a more debilitating emotional reaction than a more placid, unresponsive person. In the analysis of a task and in the design of learning, one should pay attention to such factors and design different (less demanding) acquisition paths for the more emotionally reactive learners.

Apart from the inescapable fact of life that emotions and affect are an integral part of what we call human nature, what we recognize as the constitutive aspects of our humanity, one can argue that an affectless learner would not learn much. Such a conclusion arises out of a brief consideration of the relation between emotion and motivation. Research in so-called "motivational" variables has fallen upon hard times in recent years. One reason for this is that the new cognitive orthodoxy has not yet found a reasonable mechanism for incorporating motivation into its theoretical superstructure.[9] One possible direction for this research would involve more extensive use of emotional states for motivational arguments. Suggestions proposed over the years have been essentially hedonistic ones—for example, people prefer to generate positive subjective (emotional) states and avoid negative ones. A corollary is that conditions (situations, thoughts, actions) that give rise to positive states will be sought out or repeated, and those that generate negative states will be avoided. Such a neobehaviorist position is probably the best we can do at present. At the descriptive level, it accounts for such phenomena as students' aversion to mathematics as a result of early unhappy experiences. It also brings the emotional domain into the important area of students' beliefs about strategies and tactics. Schoenfeld (1985; see also Silver, in the same volume) has argued persuasively that beliefs and metacognitions are important contributors to problem-solving performance. The extent to which such beliefs are confirmed or disconfirmed will, in turn, generate positive or negative emotional states, which will then feed back to the degree to which these beliefs may be maintained or discarded.

It is beyond the scope of this chapter to discuss the social and cultural factors that determine one's attitudes and beliefs about problem solving. I do want to recognize the importance of such factors, because they determine many of our expectations about: situations and tasks, about the social interactions that are important in the learning and teaching situations, and about our successes and failures and their relevance to our social functions and our self-image.

The argument has been advanced (e.g., Toda, 1982) that our emotions evolved to fit an environment that was wild, dangerous, and uncivilized. It is argued that we now live in a different environment, but with emotions that are outmoded and fit different situations. Wald (1978), for example, has averred that large portions of our animal heritage "have become inappropriate to civilized life" (p. 277). Wald focuses on the "violent" emotions, which he lists as pain, fear, and rage, which he considers "out of place," but he forgets other violent emotions, such as the positive passions of lust, love, ecstasy, and joy. Are these, too, out of place?

What is often forgotten is that the environment to which we originally "adapted" by developing these emotions was designed by nature, chance, or whatever; however, the very environment for which we are said to be unfit emotionally was designed by, guess who, the organism that does not fit it. I wonder whether an argument that says that it is unlikely that cultural evolution would design an environment precisely unfit for the designer is not just as viable as the argument that says we are, in fact, unfit for the present environment. Besides, in what way are we unfit?

Both affect and learning are characteristics of contemporary mankind. As psychologists, we wish to understand them, and, whenever possible, modify and make adjustments to them in order to improve the human condition. We want learning to be painless and fast, and affect to be joyful and occur when wanted. We cannot have either of these states. The best we can do at present is to understand how learning and affect come about, how they interact, and how their inevitable symbiosis can be put to the use of our students and our society.

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Footnotes

[1]The discussion will necessarily be a very sketchy and inadequate presentation of the theory. It is intended to introduce some of the notions I need to use in the following sections.

[2]For an extensive discussion of classes of values, including the problem of completion, see *Mind and Body* (1984).

[3]The manifest anxiety scale, developed by Janet Taylor Spence (Taylor, 1953), made its appearance at about the same time and had a much greater effect on ongoing covariational research.

[4]Other important contributions to error theory are Reason's papers (e.g., 1977, 1979).

[5]There are other claims about the by-products of errorless discrimination learning that need not concern us here.

[6]I have discussed these developments and their obvious relation to interruption theory in *Mind and Emotion* (1975).

[7]In my only excursion into animal experimentation, I demonstrated that the absence of reinforcement is a very powerful inducer of extreme emotional reactions even when the animals are satiated—that is, the very lack of opportunity for engaging in well-learned behavior has marked emotional consequences. When substitute behaviors are made available to the animals, however, the emotional behavior can be suppressed (Mandler & Watson, 1966).

[8]See my book *Mind and Emotion* (1975) for a more detailed discussion of the learning and affective consequences of partial reinforcement.

[9]See Gallistel (1980) for one structural approach to motivation.

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