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Chapter 6

Patterns of Experience and the Language of Mathematics

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The examination of the quantitative proficiency of minority students provides a window on the difficulties many young people have with the way mathematics is currently taught in the schools. In truth, it provides a more drastic picture of those aspects of mathematics which Underhill (1985) clearly describes as evidence that "we are teaching many mathematical concepts and skills that many learners cannot grasp and some major ones that most of the learners in many classrooms cannot understand" (p. 2). The Carnegie Forum's report (1986) on the condition of American education strongly states that "it would be fatal to assume that America can succeed if only a portion of our school children succeed."

Demographically, this report also points out that by the year 2000, one out of every three Americans will be a member of a minority group. The success of these children with the educational process, and especially in the fields of mathematics and science, is clearly a critical and crucial problem. We address two issues that affect the minority student in significant ways: The role of culturally patterned activities in children's cognitive development, and the languages of mathematics.

THE PATTERNS OF EARLY EXPERIENCE AND CULTURAL INFLUENCES ON NUMERICAL DEVELOPMENT

Influences of Family Life and Environment on Learning Style

In every culture, children engage in activities that have an underlying logic for numerical and visual-spatial relations. Many of these numerical referents are accessed differently because of cultural patterns. For instance, members of rural communities have a different and often more contextually relevant sense of space and distance than those raised in urban majority culture (Bland, 1974; Connelly, 1979; Wolcott, 1982). Seasons and cycles guide the structure of activities, rather than the clock-time that dominates most of our lives. Some of the patterns of family life in minority cultures are also quite different from that of the majority. The socialization process for Hispanic children, as well as native Americans, is one of cooperation and sharing rather than competition with the focus on individual achievement commonly seen in the majority culture. Delgado-Gaitan and Trueba (1985) have pointed out that the cooperative attitudes developed in the homes of the Hispanic communities they studied directly affected the children's understanding of "shared work." Although their research also found that the specific Mexican-American children in their study were highly versatile in integrating the values of the home with those of the school, this is not always the case. Many native American children, for example, have great difficulty with the direct questioning techniques employed in most school situations. Philips' research on the Warm Spring Reservation (1972) emphasized that native American children preferred to interact with other children rather than with adults. This preference for quiet peer interaction is underscored by many native American parents who also are uneasy in parent/teacher or parent/school meetings, especially if the topic under consideration is in any way controversial or confrontational. Direct questioning techniques rarely yield answers that truly reflect the thinking or feelings of these parents.

Learning by doing (direct experience) and careful observation of peers and elders at work is a common practice among native American children. In a similar vein, native American children have strong visual-spatial skills and they gain significantly from learning experiences that utilize their observational strength (John-Steiner & Ostereich, 1975). These children's relatively high performance on drawings (in contrast to non-native American children) provides further support for the role of graphic skills as a significant channel of communication and representation. Mathematics programs for young children that encourage the use of drawing the results of numerical experiences (such as weighing and measuring) have been a highly successful means of teaching mathematical representation.

The research of Au (1980) shows the importance of understanding the participant structure of verbal interactions in ethnic minority children's lives. Hawaiian children in the Kamehameha Early Education Program (KEEP) were quite successful in early reading tasks through the use of informal group interaction; this group interaction allowed and encouraged the children to interrupt the lesson when they had something to contribute to what was being discussed, either by the teacher or another child. Au points out that this informal interaction has been educationally successful for children with Hawaiian backgrounds because it resembles the culturally patterned story-telling practices of Hawaiian adults.

Valverde (1984) cites the work of Casteneda and Ramirez on cognitive styles of Hispanic children, which proposes a specific learning style that is more field sensitive than that of Anglo children. The sources of such holistic organization are linked, by these researchers, to large families and children's need to work together in their community setting. DeAvila (this volume) describes highly successful work in mathematics on the part of Hispanic children who were encouraged to work cooperatively in small groups, interacting verbally with each other and with effective physical materials, as well as with the teacher. Working together rather than individually significantly enhances the achievement of these children. Cultural teaching/learning practices vary not only from ethnic group to ethnic group, but also within groups, so care must be taken not to assume that practices that work well with one group of children will necessarily benefit and be successful with another. Careful testing of culturally sensitive teaching methods are needed before they can be widely adapted.

Games, Play, and Shared Activities

Many of the basic understandings and use of number that children have developed before entering school stem from games and recreational activities indigenous to the culture. It is doubtful that there is any ethnic group that does not have a multitude of childhood games and activities that, if not number oriented, at least depend upon numbers for scoring. KUI TATK, a newsletter of the Native American Science Education Association, carried an insert depicting the "Mathematics of Native American Games of Chance" complete with a map showing which native American groups enjoy these games and regional variations of them (Winter issue, 1986). This publication features articles on learning styles of native American children and is of immediate value to teachers of these children who are not from the majority culture.

It is also true that the young child who explores the roundness of a ball or the varied shapes of building blocks is experiencing one of his/her first geometric discoveries, underscoring the direct, hands-on manipulation approach to learning that characterizes young children of all cultures. Besides the more organized use of numbers in games, children everywhere, in play and in their contributions to family activities, combine, take part, share, and make comparisons. Although oversimplified, these activities certainly constitute an early view of mathematics and provide for a practical sense of the use of numbers in the everyday, "real" world. Children's participation in the family routines of work constitutes, in many cultures, a basis for survival and subtly underlines many important fundamental concepts of mathematics. The sorting of harvest crops in some native American communities certainly contributes to the children's ability to categorize and classify in the early stages of representational development. It has been stated that Hopis grow 24 varieties of corn and 23 varieties of beans, and of the 150 species of wild plants available on the mesas and desert floors, the Hopi use 134 of them in their daily life (Kennard, 1979). The direct experience Hopi children have with this sorting of the harvest and estimating quantities provides a strong background for some very basic mathematical concepts.

Farming communities and families in general provide rich experiences with spatial concepts and estimating; of distance, such as depths for planting seed and harvest yield: "make the hole about so deep;" "put about 10 seeds in each hole;" "this sack of seed will plant the entire field," are examples of relevant adult directives. Children who have direct, extensive experience with activities such as these seem to acquire an overall "sense" of number that enhances their basic understanding. And considering the emphasis that the National Council of Teachers of Mathematics (1980) is putting on the area of estimation and approximation, children who grow up with a strong background in the use of these two concepts can benefit from their informal experiences. One example of bringing formal and informal experiences is provided by a Hopi teacher (Honyouti, personal communication, 1984). He understood the strengths of his students in estimation skills and built upon these to such an extent that these native American learners did outstanding work in mathematics even after they had completed their schooling on the reservation. These learners experienced a gradual shift from informal mathematical thinking to the more formal approaches demanded by text books described by Ginsburg and Russell (1981).

In many cultural and ethnic groups, children's contribution to the daily subsistence chores provides equally rich and fundamental experiences that establish basic understandings of mathematical relationships, and provide the scaffolding necessary for more formal study. The work of Ginsburg and Russell (1981) supports the theory that children from all ethnic and language minorities enter school with good backgrounds, but schooling alters this.

Making Connections Between Formal and Informal Learning Experiences

Children everywhere experience the difficulty of making connections between learning informally, in the context of the home situation, and in the decontextualized learning environment of school. However, such a difficulty of bridging their own childhood understanding and use of number to the study of formal school arithmetic seems to be particularly serious for minority children (Cole, Gay, Glick, & Sharp, 1971; Gay & Cole, 1967; Saxe, this volume). Scott (1986) cites work in the field of "ethnomathematics" by Hunting (1985), in which Hunting proposed a research program aimed at identifying activities and processes employed in other cultures that "have potential for connections with mathematical concepts, techniques and procedures."

Thus, the use of these practical strategies when solving problems seems to help the child make sense of the very specific world and vocabulary of mathematics encountered in the classroom. Leap (this volume) cites the "practical approach" as one of the strategies used by the Ute children in his study. Ginsburg (1977) states that failure to help children make these "practical" connections is one of the serious defects in the teaching of elementary arithmetic and mathematics in the schools. Others, by contrast, point out that "practical" language often does not map onto the "technical" language of mathematics, and, therefore such learning strategies of applying home experience to school problems is less than satisfactory. Clearly such learning styles need to be meshed with concrete experiences that help map the mathematics/language relationships (Spanos et al., this volume).

How children understand numbers, whether in the context of informal learning or the more formal mathematics of the classroom, hinges very critically on language—their own and the highly specific language of the subject itself. Therefore, because language is the critical mediator of concept formation and concept development, we devote the second part of our chapter to this important concept.

THE LANGUAGE OF MATHEMATICS

Mathematics speaks to children in many different ways. It "speaks" through its very specific vocabulary, of course, but also through the broad and varied types of numerical problem solving placed before the student: from very simple counting to the more formal concepts and relationships of arithmetic, algebra, geometry, and other specialized branches of mathematics. Children acquire a variety of spontaneous concepts through home activity and playful encounters with their peers. However, it is difficult to make the connection with decontextualized learning without sensitive assistance, a point make by Cocking and Chipman (this volume). In our discussion of the languages of mathematics, we limit ourselves to these areas of mathematics encountered by preschool and elementary age children, namely, basic arithmetic.

Counting and the Count-Word Sequence

Young children's first experiences with numbers seem to stem from learning to enumerate; both the count-word sequences and the understanding of one-to-one matching relationships between the word and the objects being counted. Children everywhere struggle to acquire these two very basic concepts, and those children who enter school with both the count-word sequence and the cognitive understanding of one-to-one correspondence and the concept of number clearly have an advantage when confronted with the direct instructional sequence of formal school.

However, for those children whose basic experience with number may have been with a counting system that is unlike that of majority students (and school), just making the shift to the formal system can be a monumental conceptual task. Saxe (this volume) carefully explains the Oksapmin body-part counting method, and the efforts made by these children in school to adapt that sýstem to school arithmetic with varying degrees of success. He also cites the linguistic regularity of Japanese numeration that cannot help but support the young Japanese child's understanding of the structure of the base-10 system of numeration. Children may learn other count-word sequences in their native language that might have been created, historically, around a base system other than 10. Some native American languages, as well as others, linguistically indicate this occurrence, and there is considerable literature to indicate that native American, as well as black children, have an inordinately difficult time with understanding symbolic numeration, perhaps for this language-related reason. It is unlikely that these children immediately profit from direct instruction and lessons of the majority classroom without some carefully thought-out experiences that help to span this gap of language discontinuities. English names for teen numbers certainly do nothing to clarify the child's understanding of "10" as "one group of ten and no extra units, 11 as one group of ten and one unit," and so forth. Further, the verbalization of these teen numbers adds a further dimension of difficulty to the problem of writing these numbers correctly. Quite logically, the young child could (and often does) deduce that "thirteen" should be written with the "3" first! These types of errors are not restricted to children of language minorities but are seen frequently in primary classrooms nationwide. Poems and rhymes may help children become familiar with different ways of expressing numbers: "4 and 20 blackbirds," "four score. . . ." Although we do not either speak or count in this manner, familiarity with such expressions (and their meanings) could conceivably help some children. Another part of the challenge of conceptualizing numerical relationships is that of transforming the child's experience with spatial relationships into the sequential languages of counting and of syntax.

Arithmetic and the Concept of Number

Children facing direct instruction as school beginners must understand the count-word sequence and one-to-one correspondence as well as the meaning of "numberness;" what is "three," "four," "five" and so on? Piaget has left us with an incredibly rich background of the child's concept of number and especially the developmental growth of two critical understandings: ordinal and cardinal number. The assumption that if a young child can count, he/she must certainly understand "twoness," "threeness," and so forth, is shattered daily in classrooms everywhere.

One of the authors recently watched a 4-year-old, obviously very bright and capable, when asked to count 10 pairs of scissors to be returned to another classroom; this student carefully counted "1" (placing a pair of scissors to the side), "2" (placing another pair with the first), "3" (again putting a third pair with the others) and when reaching "10," with 10 pairs of scissors carefully placed together, went right on: "11" (placing another pair of scissors), "12," and so on. His teacher quietly placed a hand over his, and said, "No, dear, TEN pairs of scissors. Let's start again." The child repeated his performance, this time reaching "14" before being stopped and told to start again. The third time, when he reached 10 and again continued, the teacher suggested he stop and go play, turned to her colleagues and wondered aloud, "How can he count with a one-to-one correspondence and not know when he reaches 10?" It did not occur to any of the adults present to use their two hands, with fingers extended on the table, and ask the child to lay out as many scissors as there were fingers, in a direct one-to-one physical matching as an experience with quantity. His previous experience was obviously restricted to rote counting with a need for each word to match an object being counted.

Herscovics (in Steffe, Von Glasersfeld, Richards, & Cobb, 1983) comments that teachers working with young children need "to be jolted back into awareness of the difficulties in learning the number concept" (p. 13). He cites a teacher-training experiment in which the first 10 numbers in English were replaced by the words "pauve, petit, patineur, prends, patience, pour, pouvoir, patiner, plus, paisiblement." He describes the teacher reaction by indicating that the semantic content of the sentence made for quick memorization; however, the use of the words for simple arithmetic tasks produced great difficulties. Counting backwards was exceedingly hard work, and trivial addition facts became major problems requiring counting on the fingers while repeating the words several times. Herscovics states "that experiences such as these indicate the need in didactics to go beyond a purely mathematical analysis of a concept and to consider it from an epistemological point of view" (p. 13). The conclusion is that children's logic follows a different line from mature adult taxonomy. Only by weaving together how the child conceives number (namely, the way the child structures base-10 numbers) with the scientific concepts of adult can the teacher fully assist the young learner. Teachers, indeed, need to know more about children's developing epistemologies.

We stated previously that Leap (this volume) cites an often used strategy by Ute children of reverting to practical processes when confronted by a problem involving an operation with which they had only limited experience (i.e., multiplication and division). Somehow, encouragement of such a strategy, which seems so logical for all children, is limited. Even in simple arithmetic, a strict interpretation of the vocabulary of the subject, as well as use of the accompanying correct symbolic representation, is what is asked of and expected of children right from the beginning. As Ginsburg (1977) suggests, we need to make a point of helping children relate the world of school arithmetic to the real world of numbers that makes up the background with which they enter the classroom. It is critical that children have the opportunity to develop the cognitive uses of language, the language of school and specifically mathematics, which combines the codification of *spontaneous concepts* and *scientific concepts*. The latter is acquired in formal learning situations and is dependent on the more academic form of language (Vygotsky, 1962). The specific vocabulary and symbolic representation required in the study of school arithmetic must be acquired, but perhaps a more carefully guided process of acquisition, which includes, initially, the use of "practical processes" and language as well as child-invented algorithms might help bridge this immense gap.

Many children, and especially minority children, seem to go through the elementary years feeling that numbers "control" them, instead of their having power over numbers! And with such a background, it is understandable that they shy away from the advanced mathematics of high school and beyond. In the case of school arithmetic, it would seem that there is definitely a subordination of learning to teaching; a priority of subject matter over how children learn.

THE SHIFT FROM TEACHING TO LEARNING

Instead of using the valuable knowledge we already possess about how children learn spontaneously, with appropriate guidance and verbal interaction, emphasis is still being put on skills proficiency. Such proficiency is taught in an inflexible sequence accentuating drill and practice rather than developing the underlying cognitive processes that guide the acquisition of number concepts.

Children simply do not learn the same things in the same way, even within the same cultural background. Specifically in the domain of mathematics, children construct their understanding of number over many years, slowly and carefully, fitting new experiences into current cognitive structures. Many children, and especially those from ethnic minorities, often misinterpret what is being asked of them by teachers (see Mestre, this volume). Steffe et al. (1983) have emphasized the need for teachers and researchers to watch and see what it is that children do with the tasks asked of them in order to determine the intentions and understandings of the child, clearly the subordination of teaching to learning.

If the shift from teaching to learning is to occur (and the need for this shift is underscored in the report of the Carnegie Forum), the role of the teacher will drastically change. Instead of being a *director*, the teacher of young children will become an *orchestrator* of appropriate experiences, with physical materials and language. The teacher will be more of a catalyst to learning rather than a *presenter* of a body of facts and figures. Teachers who have already assumed such a role have become very effective "kid watchers," using these careful observations of children interacting with materials, their peers, and their teachers, as cues and clues to concept formation and the need for planning the "next step."

The learning of mathematics is a complex process for all children. Successful study of this important discipline must take into consideration the many and varied background experiences with which children come to school. Strengths of

learning through the visual, observational mode, and the tactile, manipulative mode need to be utilized. Some children may need direct instruction, whereas others may not have engaged in this type of verbal interaction. Some children learn better in small groups or pairs, but others thrive in more self-determined learning situations, and there are also some children who do profit from the traditional large group instructional setting.

If *all* children, regardless of ethnic origin or language background, are to be successful in the skills of mathematics, we must be prepared to support differing learning styles and a more flexible curriculum content. For the minority student, this means that classroom experiences need to be beyond textbook-based instruction. Hands-on, experiential learning, critical to the learning of all young children, needs to be structured to reflect cultural practices in the child's home and neighborhood, emphasizing interactional routines that characterize different community practices.

The challenge of mathematical thinking and its availability to children is beautifully rendered by the educational philosopher, David Hawkins (in prep.). He tells the following story:

> There is a charming story about a waif from the streets of London whose young friend and mentor finds in her a precocious interest in numbers. After a considerable time of involvement with the subject she reveals a deep concern about the nature of numbers, constructing a remarkable model to explain them. Casting the shadow of some familiar object on a screen, she then insists on a cardboard replica of the shadow. Turned on edge, its shadow is a thin straight line segment. This in its turn is represented by a thin stick, which when held normal to the screen casts a small dot-shadow. This, she announces, is how Mr. God makes numbers. All the complexity of real objects is reduced by this three-fold shadowing to their least common character, each distinguishable from the others but counting nevertheless, counting as one among a number. (p. 50) The rules of matching and counting, abstracted from our dealings with nature, yield more than the specific items or experience which went into the abstracting of them. The system of number standards and of procedures for generating new ones has, therefore, the nature of a world which transcends our knowledge of it and even our conjectures about it: complex, surprising, and inexhaustible. (p. 99)

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