Review Sheet for Final Exam

M27 – Linear Algebra Swarthmore College Prof. Scott Cook February 26, 2013

- This is a work in progress. It may be amended at any moment, up until the start of the exam. This list is NOT COMPLETE. Any topic covered in lecture or homework is exam eligible unless otherwise explicitly ruled out here. If a topic has been omitted here and you would like to inquire of its exam eligibility, please post to the forum for this exam on moodle.
- A high-efficiency study method is to rework old homework problems, trying to forget what you did before. Though the exam is unlikely to have a problem from homework verbatim, if I thought a certain style of problem was important enough to assign once, there is a better than random chance that I will think it is still important at exam time.

As stated in the syllabus

"Exams will be mostly free response and will contain some purely conceptual problems, some mostly computational problems, and some problems which require substantial conceptual **and** computational facility."

"You may use scientific (not graphing) calculators on exams. Nothing that can handle matrices."

I have posted exam 1 from last semester's class. I expect to write an exam that is similar *in format*. Do *not* expect problem that are similar *in content*.

Chapter 1

Topics to know

- Word problems - Turn the description of a system into mathematical expression(s)
- Solve a system of n equations involving m variables using matrices and row operations
- Determine all solutions of a system. How many are there? (0,1,a lines worth/1 free variable, a planes worth/2 free variables, etc).
- What is rref(A)? (state definition)
- Given A, find rref(A) and use it to determine whether $A\vec{x} = \vec{b}$ must have a unique solution for any $\vec{b}$. If not, determine how the number of solutions to $A\vec{x} = \vec{b}$ could vary between different $\vec{b}$ (for example, perhaps some $\vec{b}$ have no solution while other $\vec{b}$ have a plane's worth of solutions, etc).
- Compute rank(A).
- Basic matrix algebra operations and the geometric meaning
- Definition of a linear combination
- Describe 3 views of $A\vec{x}$

Proofs to know

- Prove basic facts about matrix algebra (such as the list of 5 I gave in lecture and you proved some of in homework).

Not eligible for the exam

- Nothing I have thought of so far. Feel free to ask.

Chapter 7

Section 7.1

Topics to know

- How to encode the rules that govern a system into a matrix (such as the lilac bush example and 7.1#52-54).
- What are eigenvectors and eigenvalues? (definition)
- What does it mean geometrically for a vector to be an eigenvector?
- Why do we care about eigenvectors and eigenvalues? (They help us know our future)
- Know that the zero vector $\vec{\mathbf{0}}$ can NOT be an eigenvector but the number 0 can be an eigenvalue. Can you think of a transformation in section 2.2 that has an eigenvector with eigenvalue 0?

Proofs to know

- Nothing I have thought of so far. Feel free to ask.

Not eligible for the exam

- Other stuff from this section (since we don't have all the tools to make complete sense of it yet)

Chapter 2

Section 2.1

Topics to know

- What does it mean for a transformation to be injective, surjective, or bijective? (state definitions)
- Determine whether a transformation has these properties.
- What does it mean for a transformation to be invertible?
- What does it mean for a transformation to be linear? (state definition - 2 properties). (recall that Bretscher approached this differently than our lecture) Be able to show whether a given transformation is linear using these 2 properties.
- Know that T is linear if and only if there is a matrix that does it. (no proof)
- If T is linear, what are the columns of the matrix that does it?

Proofs to know

- Prove that T is linear if and only if there is a matrix that does it.

Not eligible for the exam

- Nothing I have thought of so far. Feel free to ask.

Section 2.2

Topics to know

- Know the 5 types of 2 dimensional transformations we discussed. For each, know
  - i) What it does geometrically
  - ii) For all 5, that it is linear using a geometric argument (a well drawn and explained picture)
  - iii) For scaling, orthogonal projection, and reflection, show that it is linear using an algebraic argument
  - iv) What matrix does it?
  - v) I will give you the formula for orthogonal projection $\text{proj}_L(\vec{\boldsymbol{x}}) = (\vec{\boldsymbol{x}} \cdot \vec{\boldsymbol{u}})\vec{\boldsymbol{u}}$, where $\vec{\boldsymbol{u}}$ is a *unit* vector along L.
  - vi) How to use this knowledge to solve problems

Proofs to know

- See above

Not eligible for the exam

- Don't memorize the formula for orthogonal projection

Section 2.3

Topics to know

- How do we multiply two matrices? (state definition and perform the computation)
- Why do we define matrix multiplication this way? (Answer - So that composing two linear transformations is equivalent to multiplying their matrices. Get the order right; the order you write the symbols for matrix multiplication is the opposite of the order you draw the blob diagram.)
- Given 2 matrices, determine if they can be multiplied. If so, do it.

Proofs to know

- Prove that the composition of 2 linear transformations is linear (verify the 2 properties that define linearity without referring to any matrices)

Not eligible for the exam

- Proofs of theorems 2.3.5-2.3.8
- Block matrices

Section 2.4

Topics to know

- Know that a transformation T is invertible if and only if it is bijective.
- Know several ways to check if an $n \times n$ matrix is invertible.
- If we know a linear transformation is invertible, how do we find its inverse? (Answer - Find its matrix A, augment with I_n, row reduce.)

Proofs to know

- Show that $(BA)^{-1} = A^{-1}B^{-1}$ using a blob diagram

Not eligible for the exam

- p.85-88