The objective of this homework set is to prove the important fact that

$\vec{x}\cdot\vec{w} = \|\vec{x}\|\ \|\vec{w}\| \cos\theta$

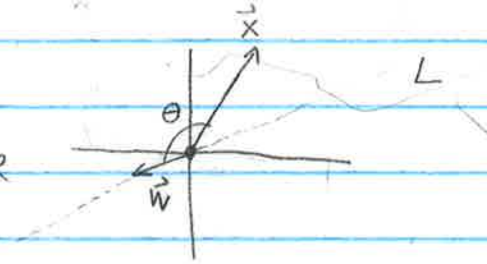
for any 2 vectors $\vec{x}$ and $\vec{w}$ in $\mathbb{R}^2$ where θ is the (smallest) angle between them (when they are rooted at the same point).

3pts a) Prove $\|k\vec{x}\| = |k|\ \|\vec{x}\|$ for all $\vec{x}\in\mathbb{R}^n$ & all scalars k

6pts b) Let L be the line thru the origin parallel to $\vec{w}$.

Consider $\vec{x}^{\parallel} = \text{proj}_L(\vec{x}) = \left(\frac{\vec{x}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\right)\vec{w}$

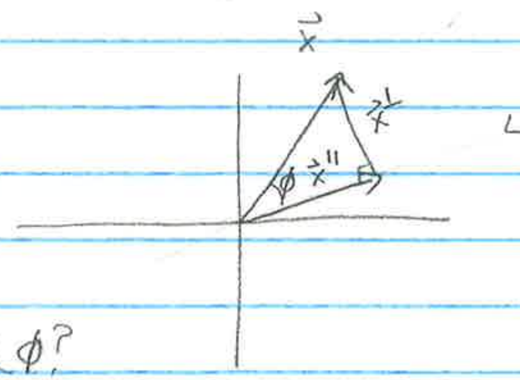
$\vec{x}^{\perp} = \vec{x} - \vec{x}^{\parallel}$



Then $\vec{x}^{\parallel}, \vec{x}^{\perp}, \vec{x}$ form a right triangle

Let ϕ be the angle between $\vec{x}$ and $\vec{x}^{\parallel}$

Note $0 \le \phi \le \pi/2$.



Show that $|\vec{x}\cdot\vec{w}| = \|\vec{x}\|\ \|\vec{w}\| \cos\phi$

(Hint: Express $\cos\phi$ using the right triangle & simplify using the definition of proj_L & part a).

2pts c) What is the relationship between θ & ϕ?

(It depends on the sign of $\vec{x}\cdot\vec{w}$)

3pts d) Show $\vec{x}\cdot\vec{w} = \|\vec{x}\|\ \|\vec{w}\| \cos\theta$ (Contrast to part b – no absolute value bars and different angle $\phi \rightsquigarrow \theta$).