Physics 114 Practice Test 2 2018

Instructions:

- Please solve these problems, putting your answer in the blank space.
- The real test will have 6 questions … this practice test has 9
- Should an answer be wrong in the real test, you will have an opportunity to correct it … by doing a calculation, open-book, showing your steps and reasoning and if you can, getting the correct answer at the end of your calculation.
- You can use your cheat sheets and a calculator.
- Good luck!

Useful data and conversion factors:

Ideal gas constant R = 8.31 J/mol/K

Boltzmann's constant: k = 1.38×10^{-23} J/K

Planck's constant: h = 6.63×10^{-34} J s

Proton mass: 1.66×10^{-27} kg

Electron mass: 9.1×10^{-31} kg

Stefan-Boltzmann constant: $\sigma = \dfrac{2\pi^2 k^4}{15h^3c^2}$

Molar volumes: Ice: 19.58 cm^3, Water: 18.02 cm^3

Latent heat of fusion for water: 6.03 kJ/mole

1 eV = 1.60×10^{-19} Joule

Absolute zero: 0K = -273.15 ^oC

speed of light: c = 3.00×10^8 m/s

Integrals of the form: $I(p) = \int_0^\infty \frac{x^{p-1}}{e^x - 1}dx$ for p>1 have the value *I(p) = ζ(p)Γ(p)* … see table below:

p	$\zeta(p)$	$\Gamma(p)$	$I = \zeta(p)\Gamma(p)$
3/2	2.612	$\sqrt{\pi}/2$	$1.306\sqrt{\pi}$
5/2	1.341	$3\sqrt{\pi}/4$	$1.006\sqrt{\pi}$
3	1.202	2	2.404
4	$\pi^4/90$	6	$\pi^4/15$
6	$\pi^6/945$	120	$8\pi^6/63$

Integrals of the form $I(p) = \int_0^\infty \frac{x^{p-1}}{e^x + 1}dx$ have the following values which may be helpful:

I(3/2) = 0.678; I(1/2) = 1.072

1. A system of N electrons (which have their usual mass and spin 1/2) exist in a nanowire of length L, which we can model as a one-dimensional box. For this problem, ignore interactions between electrons, treating them like a 1d ideal fermionic gas. What is the Fermi temperature, T_F , in degrees K, if $L = 10^{-6}$ m and $N = 10^{8}$?

Answer: ___________________

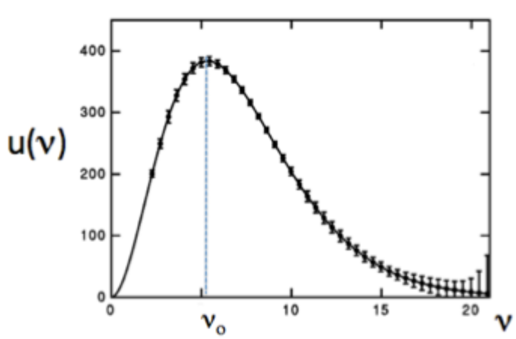
2. Photons are massless bosons with two polarization states. Their chemical potential, μ , is just what you'd expect from particles which are not conserved. If you have a box of volume V and temperature T, what is the mean number, N, of photons in the box?

Answer: ___________________

3. Consider a paramagnetic system of N spins that can take on, not two values, but *three* values: $s_i = -1, 0, 1$. The spins sit on a lattice. They do not interact with each other, but they do interact with a magnetic field B, so $E_i = -\mu_0 B s_i$ is the energy of the i[th] spin. (Here, μ_o is the magnetic moment for a single spin.) What are the free energy, F, and the average magnetization $\langle M \rangle$, of these spins at temperature T ? (Hint: Does your $\langle M \rangle$ have reasonable limits for low and high T ?)

Answers: ___________________ ; ___________________________

4. Below is the Black Body spectrum of a cosmological object, where u(ν) dν is the amount of energy per unit volume produced in the small interval dν near the frequency ν . The spectrum has a peak at frequency ν_0 . In terms of ν_0 and constants of nature, please give the approximate temperature, T, of the object. (Without Mathematica to solve a transcendental equation, it is fine to just give it approximately.)



Answer: ____________________

5. At a pressure of 1.000 atm. the freezing temperature of water is 273K. At a pressure of 100.0 atm, at what temperature will water freeze?

Answer: ____________________

6. This is a question about an ideal gas. Consider the three variables *V, N* and $<v>$ which correspond to the volume, number, and expected speed of gas molecules. Suppose you change *one* of these three variables, but you are careful to *hold the other two constant*.

Which variable(s), when changed in this way, will result in a change in pressure P?
Which variable(s), when changed in this way, will result in a change in chemical potential, μ?

Answers: ________________________ ; ________________________________

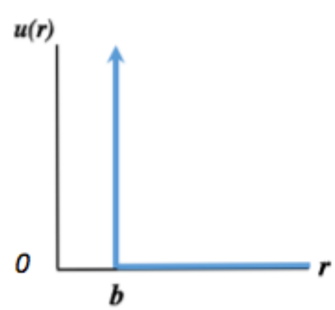
7. A system of fermions at temperature *T* has many nondegenerate energy levels. Two energy levels happen to have $\varepsilon_1 = \mu - c$ and $\varepsilon_2 = \mu + c$. Here, μ is the chemical potential and *c* is a constant. Suppose the probability of energy level 1 being occupied is calculated. Just to be concrete, say this probability has the value A:

$$n(\varepsilon_1) = \mathrm{A}$$

Please calculate the probability that energy level 2 is occupied, *giving your answer purely in terms of the value* A.

Answer: ___________________

8. A classical gas of *N* particles at temperature *T*, exists within a volume *V*. From this info, you can find the mean distance between particles, which we'll call *a*. Say that the particles interact like "hard spheres" of size *b*. In other words, if two particles are a distance *r* apart, their potential energy is described by the graph below:



Please estimate the relationship between a and b, in the case that $P = 1.10\,\rho\, kT$. In other words, find out how the two sizes a and b are related when the gas pressure is 10% greater than the pressure of an ideal gas. *(Hint: A necessary step is to calculate the second virial coefficient, B(T).)*

Answer: ____________________

9. Consider the spontaneous magnetization per spin, m, of the two-dimensional Ising model with zero external magnetic field. It's the case that $m = 0$ when $T > Tc$. But there is a continuous phase transition as temperature is lowered past Tc, and for $T \leq Tc$ the magnetization is known to be (thanks to Lars Onsager):

$$m^8 = 1 - (sinh 2J/kT)^{-4}$$

From this information, please find an expression for Tc in terms of J and k_B. Also, please find the critical exponent β which is defined as follows in the limit of small $t = (Tc - T) / Tc$ as:

$$m \sim t^{\beta}$$

Answers: _______________________ ; ______________________________

Answers:

1. $T_F = 1.1\times 10^{13}$ K

(Hint: You need to use the 1d density of states for particles in a box, and integrate over all of the k states up to k_F and set this integral equal to N. You should get $k_F = N\pi / 2L$ where the "2" in denominator comes because there will be 2 electronic spin states occupying each k state. Then you use the relationship between k_F and T_F and plug in numbers.)

2. $$N = 2.404\frac{V(kT)^3}{\pi^2 c^3 \hbar^3}$$

3. $F = -NkT \ln[1 + 2\cosh(\mu_o B/kT)]$; $\langle M\rangle = 2N\mu_o \sinh(\mu B/kT) / [1 + 2\cosh(\mu_o B/kT)]$

4. $$T = \frac{h\nu_o}{k\,2.82}$$

5. 272 K

6.

Changing any one of the three variables *will* change P.

Changing any one of the three variables *will* change μ.

7.

$$n(\varepsilon_2) = 1 - A$$

8. $a = 2.76\,b$

(In other words, the typical spacing between particles is about 3 times their diameter.)

9.

$$\frac{kT_c}{J} = \frac{2}{\ln(1+\sqrt{2})} \approx 2.269.$$ and $\beta = 1/8$