Relativistic Heat Conduction

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Outline

The Fourier Equation

- First proposed in 1807, still widely used

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u + \frac{1}{k} S(\mathbf{x}, t)$$

- $u =$ temperature, S is a heat source
- *Parabolic* PDE: coefficients satisfy parabolic formula $B^2 - AC = 0$
- e.g. in 2D: $A\frac{\partial^2 u}{\partial x^2} + 2B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + \cdots = 0$
- In heat equation, y represents time–so we have no second time derivative.
- In principle, heat starts changing at any point the moment a source is added–violates law of causality

Hyperbolic Heat Equation

- Solves problem of infinite speed of heat propagation by adding in second-order term:

$$\frac{\tau}{\alpha}\frac{\partial^2 u}{\partial t^2} + \frac{1}{\alpha}\frac{\partial u}{\partial t} = \nabla^2 u + \frac{1}{k}(S(\mathbf{x}, t) + \tau\frac{\partial S}{\partial t})$$

- Inclusion of second time derivative term means this is now *hyperbolic.*
- Feature of hyperbolic PDE's: any can be transformed to (inhomogeneous) wave equation (in this case, with a damping term $\frac{1}{\alpha}\frac{\partial u}{\partial t}$)

Issues with HHE

- Can be shown that entropy production in HHE model is given by

$$\sigma = \frac{1}{kT^2}(\mathbf{q} \cdot \mathbf{q} + \tau \mathbf{q} \cdot \frac{\partial \mathbf{q}}{\partial t}),$$

  where $\mathbf{q}$ is the heat flux density
- In certain regimes, entropy can decrease
- Requires very steep decrease in density–not certain that this scenario is possible (but if so, violation of second law)

Statement of the RHE

- $$\frac{\alpha}{C^2}\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} = \alpha \nabla^2 u + \frac{\alpha}{k} S$$
- C is speed of "second sound" (more on next slide)
- Note: as $\alpha \ll C^2$, this reduces to the Fourier equation
- Identical to sourced Maxwell equation in Lorenz Gauge–admits damped wave solutions

Speed of Second Sound

- Classically, heat spread by diffusion, dictated by thermal diffusivity α
- Quantum–additional method of heat transfer, namely by wave-like phenomena: Second sound
- In metals and rigid media, second sound is carried by phonons, relatively fast compared to diffusion ($\alpha/C^2 \ll 1$)
- In soft matter, much slower–wave term not necessarily negligible in RHE

Issues with RHE

- Second sound not widely accepted as having physical meaning
- Not really relativistic: treats second sound speed C like speed of light c without much motivation
- Are phonons real? Who's to say!
- Demonstrates that second law of thermodynamics is equivalent to law of causality (What?!)

References

1. J. A. Lpez Molina et al., Hyperbolic and relativistic heat transfer equations: a comparative analytical study Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science, Vol. 470, No. 2172. (8 December 2014), doi:10.1098/rspa.2014.0547
2. A. Barletta, E. Zanchini, Hyperbolic heat conduction and local equilibrium: a second law analysis, International Journal of Heat and Mass Transfer, Volume 40, Issue 5, 1997
3. Y.M. Ali, L.C. Zhang, Relativistic heat conduction, Int. J. Heat Mass Trans. 48 (2005) 2397.
4. https://en.wikipedia.org/wiki/Relativistic_heat_conduction