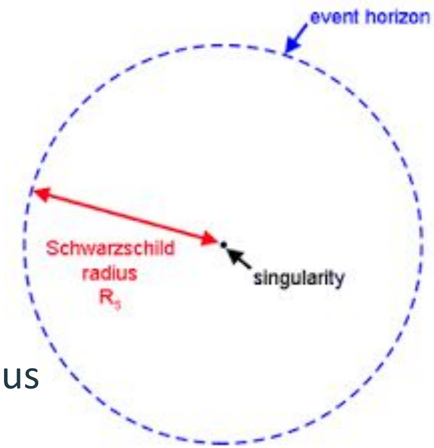
$\Delta S \geq 0$

Thermodynamics of Black Holes

Noah Lifset

What are Black Holes?



- Take a mass, squish it down to $2MG/c^2$, the Schwarzschild Radius
  - Outside of this radius, everything is “normal”
  - Inside, you’re trapped--permanently
  - THEY DON’T SUCK
- A black hole can be completely described by two parameters: mass and angular momentum
- How do we reconcile black holes, and the extremes of general relativity, with other branches of physics?

Black Holes and Entropy

- How much entropy does a black hole have? It’s just a singularity...
- It can’t have zero, or else we could violate the second law by throwing an object with entropy into it
- Applying ideas of microstates and macrostates gets tricky
  - With only two macrostate parameters, how do we determine microstates?
  - Some consider the environment around black holes, the gravity on the event horizon, or the number of states that could lead to a black hole
- However you conceptualize it, there are a few ways to derive an actual equation

Deriving Bekenstein-Hawking Entropy

- We start with Hawking radiation, which treats black holes as a black body emitter:

$$k_b T = \frac{c^3 \hbar}{8\pi G M}$$

- Next we use the first law of thermodynamics. Small changes in a black hole that do no work (no torque) will result is a small change in entropy:

$$dE = c^2 dM = T dS_{BH}$$

- Plugging in, integrating, and assuming a zero mass black hole has no entropy gives us:

$$S_{BH} = \frac{4\pi G M k}{c\hbar}$$

- Then we substitute in the Schwarzschild radius to get:

$$S_{BH} = \frac{A k c^3}{4 G \hbar}$$

The Second Law of Thermodynamics

- If area and entropy are proportional, do we have $\frac{dA}{dt} \geq 0$?
- Nope. Hawking radiation could cause the area to decrease.
- The second law is saved, though, by the entropy of the radiation itself!
    - To increase black hole entropy, stuff must fall in.
    - To decrease black hole entropy, hawking radiation must go out.