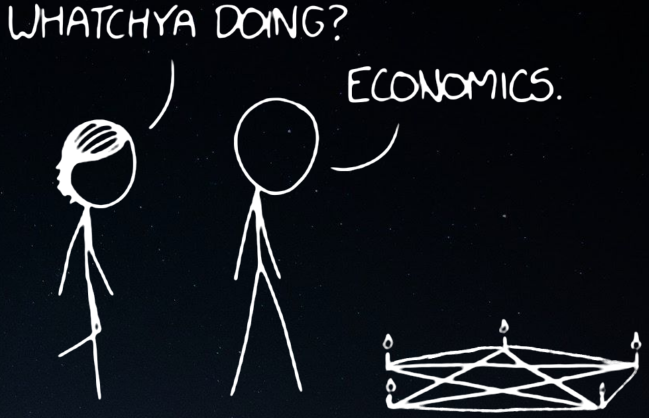
Stat-Mech & Econophysics



Matt Palmer

Background: A Cross-Disciplinary History

- Research methodology in physics has been evolving based on rationality, experiment, and a fundamental search for understanding.
- Statistical mechanics was developed in the 19th century by physicists who believed atoms to be the fundamental building blocks of matter (Maxwell, Boltzmann, Gibbs, etc.)
- Allows physicists to leverage computational power to model complex systems.
- There exists a historical overlap between mathematics, physics, and economics → Poincaré, Black-Scholes, etc.

Physics & Economics: Econophysics

- Given the availability of data, a new field has emerged ➜ *econophysics*
- Finance and economics are treated as complex systems and analyzed using physics tools
- Stochastic analysis improves with the quality of economic data

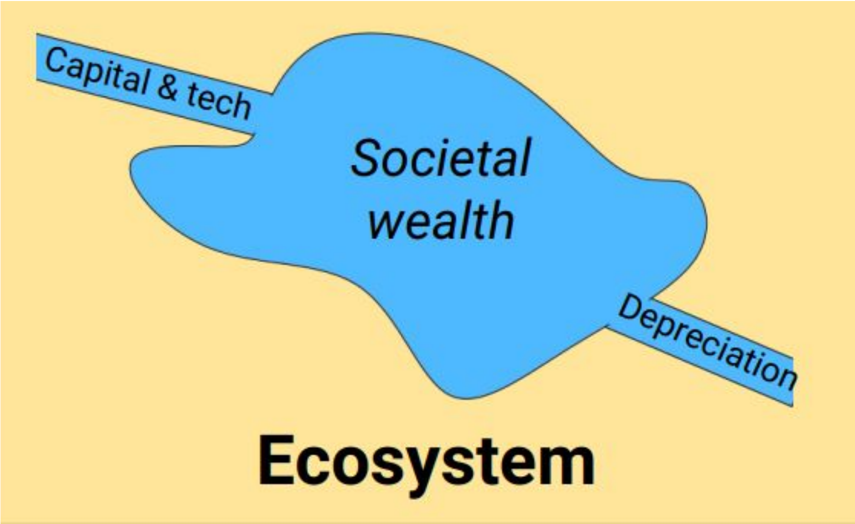
- For interdisciplinary work to be meaningful, subjects must be similar
  - Methods, thought, principles
  - Econ & physics?

Econophysics: Equilibrium & SSE

- Though economic agents are *animate* while physical systems are *inanimate*, we can reconcile the difference through Standard Economic Theory
    - By assuming “rational behavior,” we can generalize the actions of a population ➔ *utility*… Agents work to maximize their well-being.
- Thus, economists concern themselves with the time evolution of *less rigorous* complex systems.

Physics & Economics: Econophysics

- Economics → Supply & Demand
  - Market equilibrium & response to changes
- Physics centers itself around equilibria & responses to external forces
- Both concerned with stability & equilibria of isolated systems → connecting smaller systems to a larger environment.



The Boltzmann Distribution: Derivation

Divide a thermodynamic system into two (unequal) parts.

The total energy is the sum of both parts, but we know the probability to be the product of the individual probabilities...

$$\epsilon = \epsilon_1 + \epsilon_2 \quad \&$$
$$P(\epsilon) = P(\epsilon_1)P(\epsilon_2).$$

Assuming an equal probability of all microstates (and energy conservation), there is only one solution to this system!

The Boltzmann Distribution of Energy

$$P(\epsilon) \propto e^{-\epsilon/k_B T}$$

As we have seen in class, gives the probability of finding a system in a state with energy ϵ.

- A use of the *canonical ensemble*
  - Small systems coupled with a large reservoir
- Entropy is maximized and energy conserved (Laws 1 & 2)
- Only depends on statistical nature and conservation law!

Plausible: Systems governed by

1) **A conservation law**
2) **Statistics**

Can be modeled with our Boltzmann distribution.

Conservation of Wealth

Consider an economic transaction between agents *i* & *j*

$$m_i \to m_i' = m_i - \Delta m,$$
$$m_j \to m_j' = m_j + \Delta m.$$

$$m_i + m_j = m_i' + m_j'.$$

When *i* pays money to *j* for goods or services, the balance changes accordingly... Thus the total amount before and after the transaction is equivalent & conservation holds

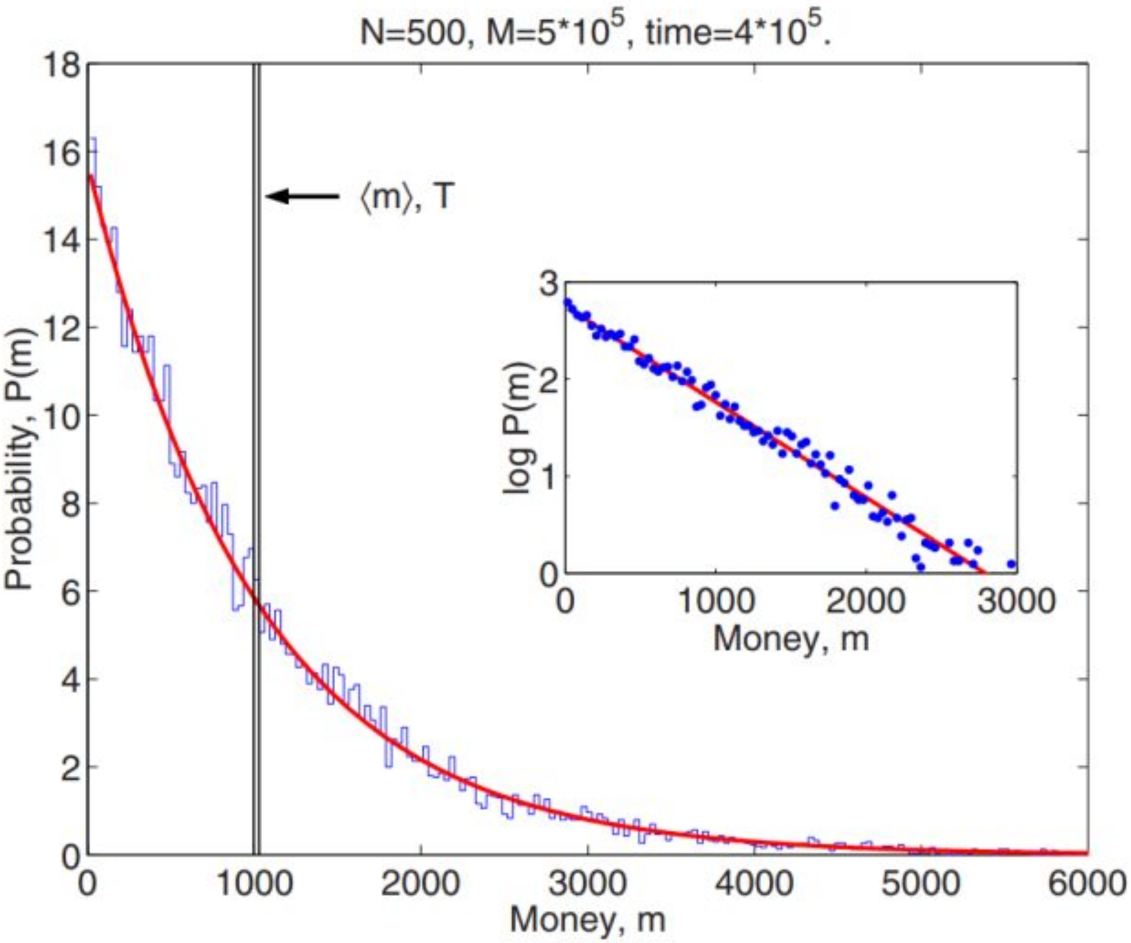
Assumptions: Wealth Conservation

- Money is conserved, goods and services are not... They may enter and exit the SSE, e.g. Haircuts occur spontaneously
- The role of central banks ➔ apparently break conservation law?
  - No! Rather, analogous to energy influx. Start with an idealization of a closed system & generalize!
- Local conservation **key** to a successful economy! (less inflation)

The Boltzmann Distribution of Money

$$P(m) \propto e^{-m/T_m}$$

- T_m is the *money temperature* or average amount of money per agent.
- Researchers use agent-based simulations of transfer between agents
  - Random pairs transfer some amount of money.
  - Distribution first broadens to a symmetric Gaussian curve, characteristic for a diffusion process.
  - Pileup occurs around m=$0, an exponential shape results



The histogram and points are the stationary probability distribution of money. *P(m)* obtained in computer simulations with $\Delta m = \$1$.

The solid curve is a fit to the Boltzmann law.

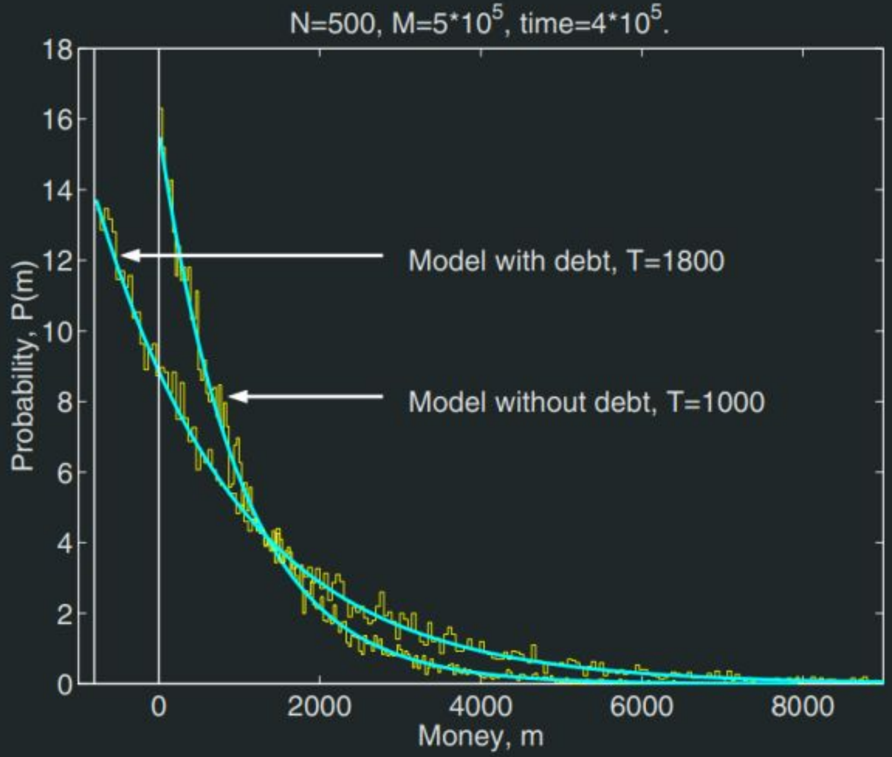
The vertical line is the initial distribution of money m = $1,000.

From Dragulescu and Yakovenko

Debt

A more accurate model would include debt, i.e. negative money...

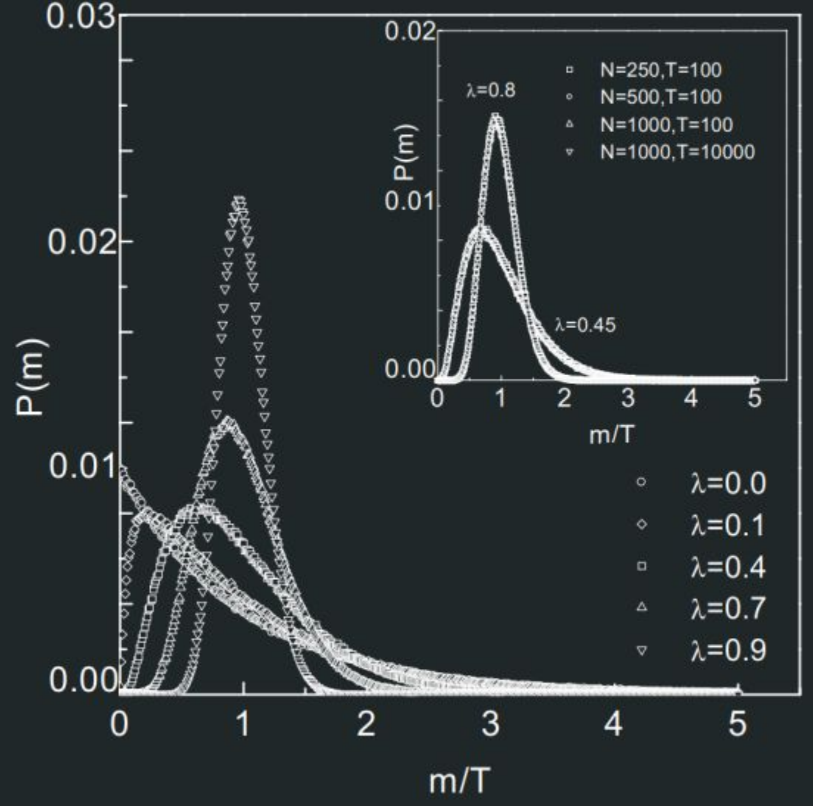
- No barrier at *m=0.*
- Limited to $800, like a barrier to a loan.
- Similar exponential behavior, but with new boundary conditions.



Savings

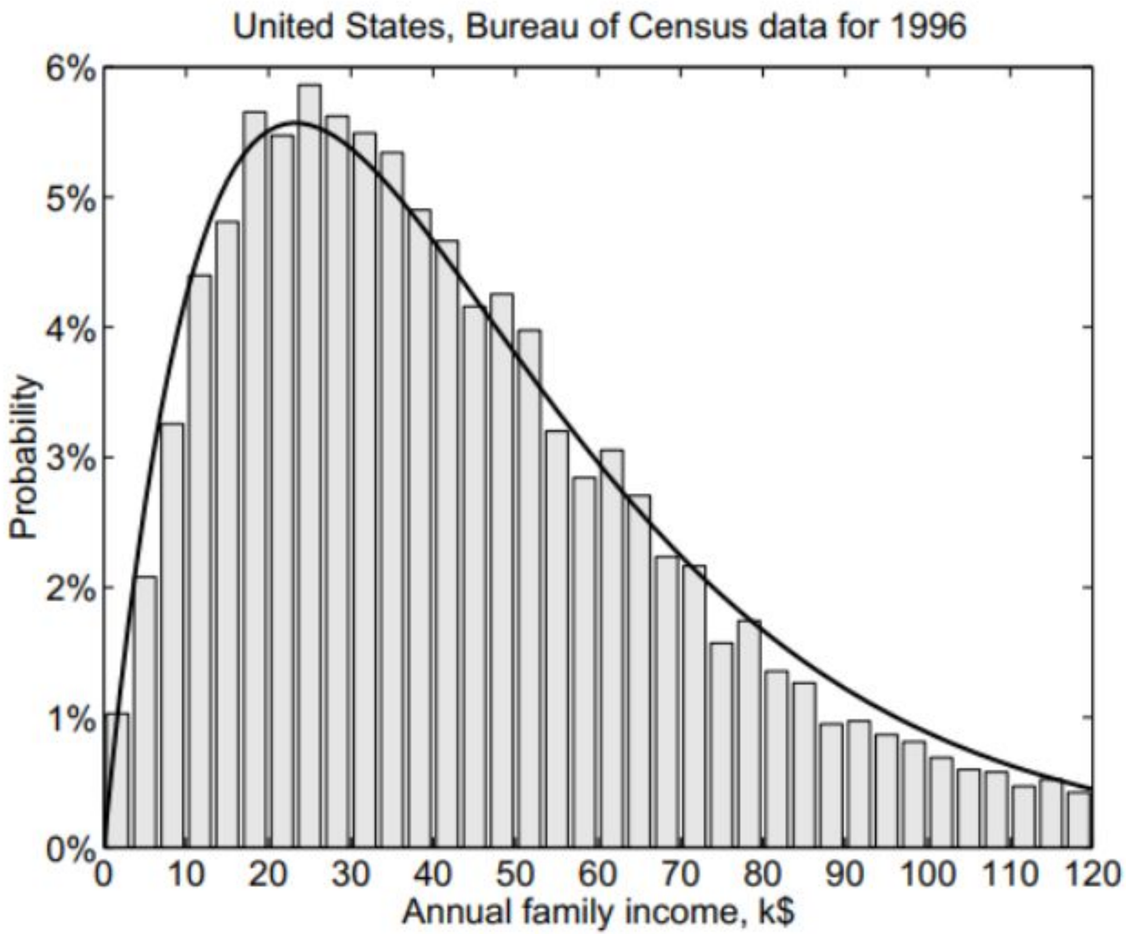
A further accurate simulation would consider that agents save some of their money! *Chakraborti & Chakrabarti* consider such a model.

- Based upon MPS ➔ generalize to population.
- As we increase MPS, the form becomes more familiar as a Boltzmann distribution!



Is this behavior observed in practical settings?

Luckily, increased data availability makes testing such a distribution possible!



Assuming incomes within households are uncorrelated (as data shows), the income PDF for households (two earners) can be modeled:

$$P_2(r) = \int_0^r P_1(r')P_1(r - r')dr' = \frac{r}{R^2}e^{-r/R},$$

thus, theoretical results described by the Boltzmann law can be used to model recent income data.

Physics has a lot to offer to other disciplines.

- Many soft subjects and social sciences do not yet have the analytic toolbox that physics employs.
- With large amounts of data now available (thanks internet), researchers need a way to sort through data and model trends.
- Physics presents a number of ways to model dynamic and stochastic (stat-mech) systems.
- Stat-mech can help economists solve issues, but applications extend to other domains.

Other Applications: Derivatives

- A *derivative* is a financial contract between two parties, where the value of the payoff is derived from another instrument.
  - Can be used as insurance or to magnify gains
  - Extremely complex & interconnected
    - Use econophysics to construct network models and identify breakdowns in opaque markets!

Other Applications: Financial Crises

- Economic crises seem to pop up every few years or so, with economists and the public alike taken by surprise.
  - Most economic models are elegant, but idealistic
  - Economists are over-reliant on rational behavior, and thus oblivious to certain bubbles.
    - Stochastic models leveraging stat-mech might be able to better replicate the financial market & allow economists to identify concerning sectors!

References

[1] Eder Johnson de Area Leo Pereira, Marcus Fernandes da Silva, and H.B.B. Pereira. Econophysics: Past and present. *Physica A: Statistical Mechanics and its Applications*, 473:251 – 261, 2017.

[2] Victor M Yakovenko. Econophysics, statistical mechanics approach to. In *Encyclopedia of Complexity and Systems Science*, pages 2800–2826. Springer, 2009.

[3] H.E Stanley, L.A.N Amaral, D Canning, P Gopikrishnan, Y Lee, and Y Liu. Econophysics: Can physicists contribute to the science of economics? *Physica A: Statistical Mechanics and its Applications*, 269(1):156 – 169, 1999.

[4] PierCarlo Nicola. *Mainstream mathematical economics in the 20th century*. Springer Science & Business Media, 2013.

[5] Herman E Daly. *Steady-state economics: with new essays*. Island Press, 1991.

[6] S. Blundell and K.M. Blundell. *Concepts in Thermal Physics*. Oxford University Press, 2006.

[7] Victor M Yakovenko and J Barkley Rosser Jr. Colloquium: Statistical mechanics of money, wealth, and income. *Reviews of Modern Physics*, 81(4):1703, 2009.

[8] Adrian A Dragulescu and Victor M Yakovenko. Statistical mechanics of money. *The European Physical Journal B-Condensed Matter and Complex Systems*, 17(4):723–729, 2000.

[9] Anirban Chakraborti and Bikas K Chakrabarti. Statistical mechanics of money: how saving propensity affects its distribution. *The European Physical Journal B-Condensed Matter and Complex Systems*, 17(1):167–170, 2000.

[10] P. Krugman, R. Wells, and M.L. Olney. *Essentials of Economics*. WORTH PUBL Incorporated, 2009.

[11] Adrian A Dragulescu and Victor M Yakovenko. Statistical mechanics of money, income, and wealth: a short survey. *arXiv preprint cond-mat/0211175*, 2002.

[12] Stefano Battiston, Guido Caldarelli, Co-Pierre Georg, Robert May, and Joseph Stiglitz. Complex derivatives. *Nature Physics*, 9:123, March 2013.

[13] Thomas Lux and Frank Westerhoff. Economics crisis. *Nature Physics*, 5(1):2, 2009.

[14] Stefano DellaVigna and Ulrike Malmendier. Paying not to go to the gym. *American Economic Review*, 96(3):694–719, June 2006. doi: 10.1257/aer.96.3.694.