Collisionless Relaxation

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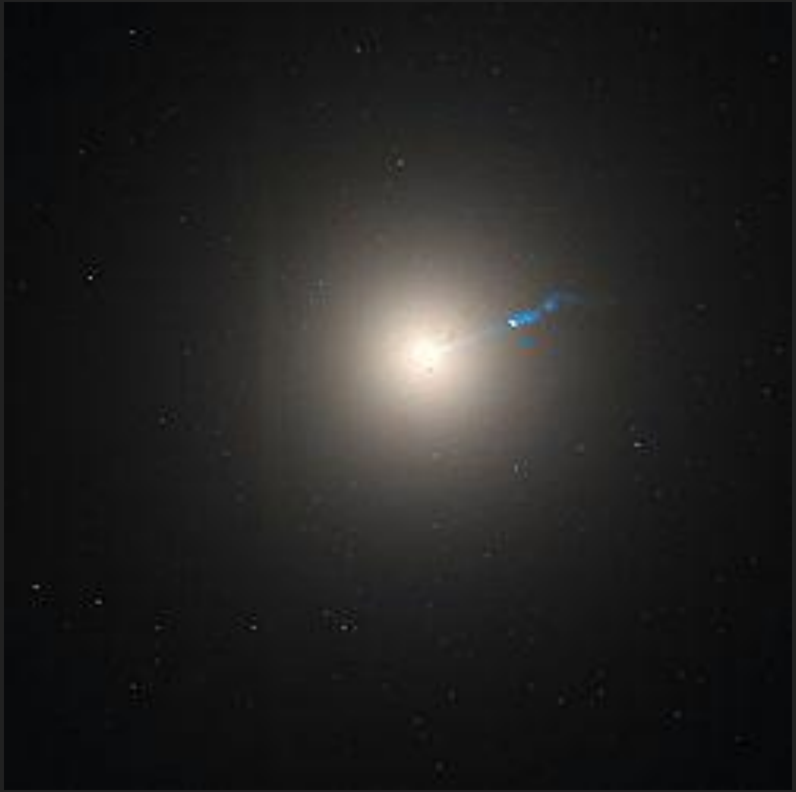
Elliptical Galaxies

- Collisionless star systems: Overall gravitational potential dominates interactions
- Regular light profiles proportional to R^¼
- All stars have about the same color

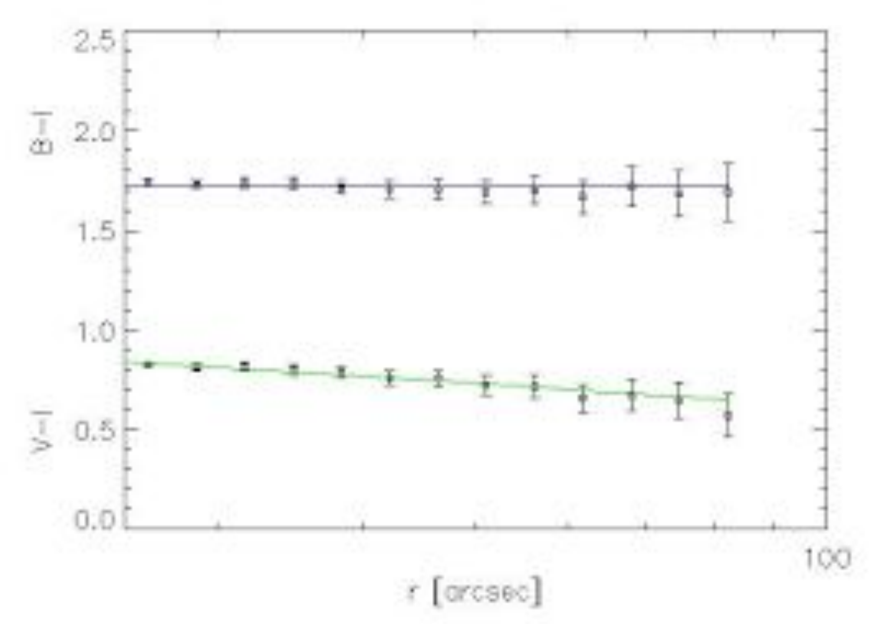


https://en.wikipedia.org/wiki/Messier_87

Observations -> Hypothesis

Need:

- Short collapse time
- No mass dependence
  - Otherwise, significant color gradients would be observed.
- Cannot relax via close interactions



Solution?

- Fluctuating potential
- Treat galaxy like a gas of non-interacting, distinguishable particles

Stat Mech analogy: mixing

- Chaotic initial state to "smooth, relaxed" equilibrium state
- Looking for a distribution function that explains time evolution
- Slow mixing in galaxies:
  - Many orbits cross narrow "shell" features
  - Most stars in the galaxy not found in these shells
  - That's good! Means the motion is chaotic
- Complex have stochastic probability distributions: we can't predict them.

https://apod.nasa.gov/apod/ap140105.html
The shells are thought to be disturbances in an otherwise smooth density distribution caused by interacting with the spiral galaxy in the upper center (points to two-body relaxation)

Real Elliptical Galaxies: Time-dependent potential

- Lynden-Bell proposes that the relaxation time scale is dependent on a fluctuating potential:

$$T_{vr} = \frac{3}{4}\left\langle\frac{\dot{\Phi}^2}{\Phi^2}\right\rangle^{-1/2}$$

- Rapidly fluctuating potential -> orbital instability -> change in particle energy
  - Spherical collapse implies all stars pass through the center at some point
- Combine this with a collisionless Boltzmann distribution
- Relabelling particle energies is a little problematic, but we go for it

Jeans Theorem

"Phase space density must be constant along trajectories at a given time"

- Starting point: how do we find a distribution function? Start with Liouville equation, which says that the distribution function stays constant as one follows the motion of any star:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} - \frac{\partial \Phi}{\partial \vec{x}}\frac{\partial f}{\partial \vec{v}} = 0$$

- Jeans theorem requires (quite problematically) that solutions for f are integrals of motion (this is problematic because they don't always work)

Even though Jean's theorem is problematic, we want to find a coarse-grained distribution function (an integral of motion), which will tell us the density of points in a small, equally partitioned cells position and momentum space.

$$F(\vec{x}, \vec{v}, t) = \frac{1}{\Delta^6 \mu_i} \int_{\Delta^6 \mu_i} f d^3\vec{x} d^3\vec{v} = \frac{n_i n}{\Delta^6 \mu_i}$$

n_i is the occupancy number in a cell with volume delta^6 mu. We want to figure out the number of microstates W that correspond to a macrostate where n particles occupy this cell:

$$W(\{n_i\}) = \frac{N!}{n_1! n_2! \ldots n_I} \prod_{i=1}^{I} \frac{\nu!}{(\nu - n_i)!}$$

From W we would be able to calculate the entropy S = lnW and find the state where S is maximized in order to find a most probable distribution function.

We have to deal with constraints on number of particles and energy. The easiest way to treat this maximization of entropy is to introduce Lagrange multipliers for N and E.

$$\delta \ln W - \lambda_1 \delta N - \lambda_2 \delta E = 0$$

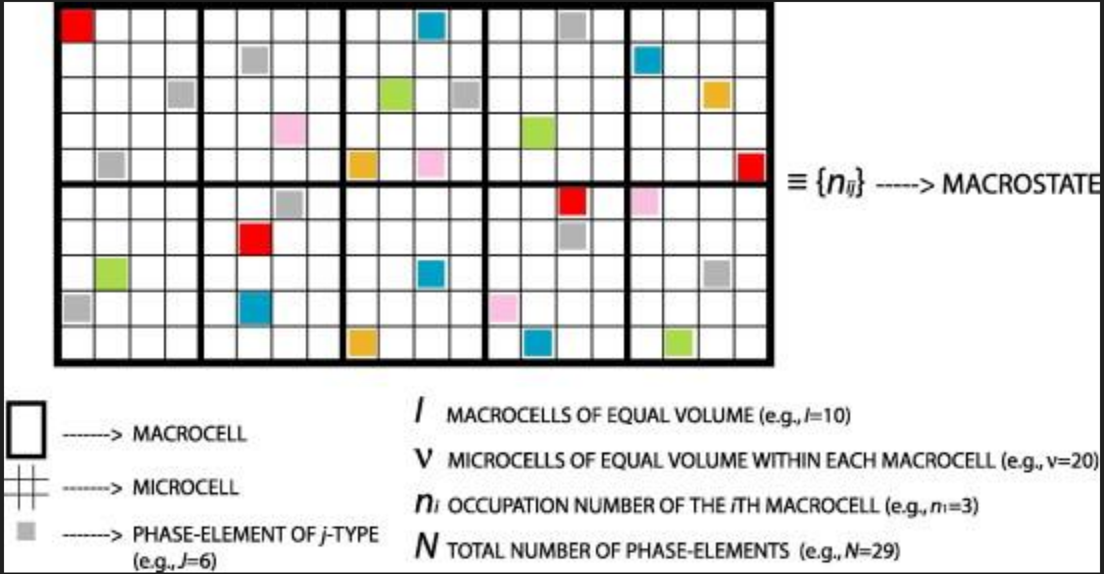
We can calculate N, the total number of particles in the system, by summing all the occupancy numbers up and using Stirling's approximation to replace ln(N!)

$$F_i = \frac{\eta n_i}{\nu}|_{S=max} = \frac{\eta}{\exp(\lambda_1 + \lambda_2 \epsilon_i) + 1}$$

Where eta = n/(d^6 mu) is the phase-space density inside each occupied element. This is gives the coarse-grained function for the ith microcell. This starts to look a little more familiar when we replace the first multiplier with mu and the second with beta:

$$F_i = \frac{\eta n_i}{\nu}|_{S=max} = \frac{\eta}{\exp(\lambda_1 + \lambda_2 \epsilon_i) + 1} = \frac{\eta \exp[-\beta[\epsilon_i - \mu]}{1 + \exp[-\beta[\epsilon_i - \mu]}$$

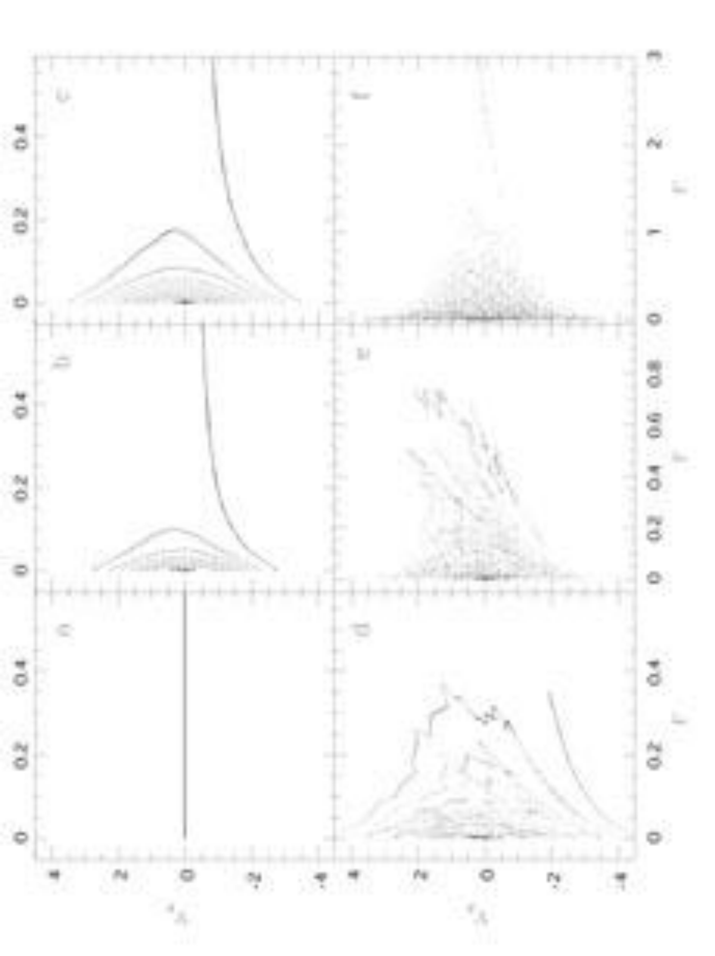
Partitioning of phase space



Particles migrate around in phase space, occupy different cells, lots of microstates that make up a bigger microstate, which make up a macrostate (fine vs coarse) (Bindoni & Secco)

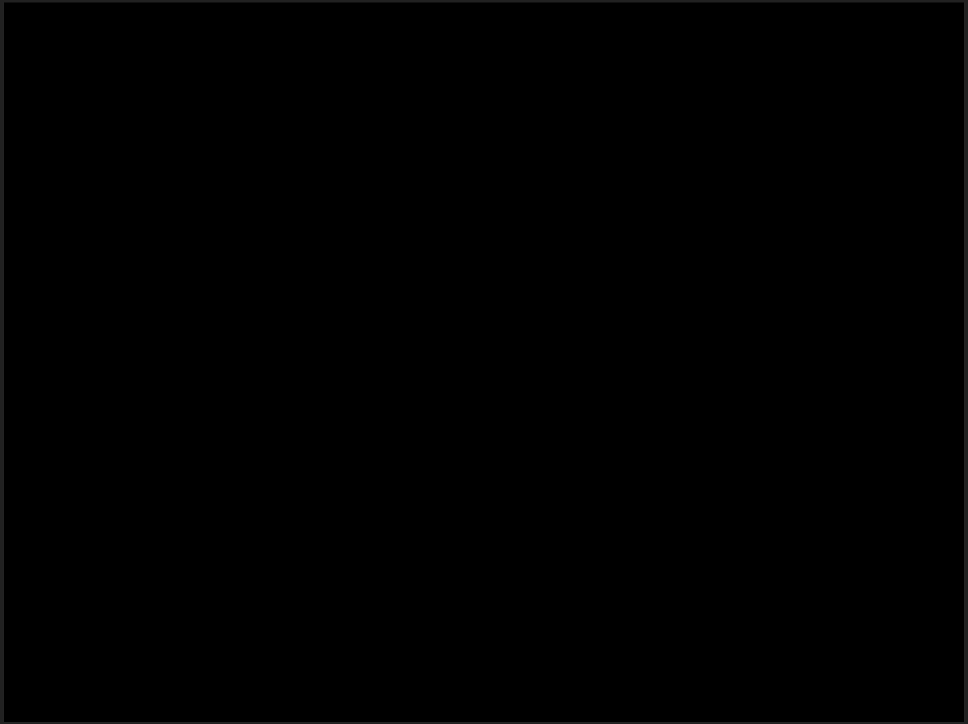
Simulation comparisons

- Phase mixing (left) vs Chaotic Mixing (right)
  - Phase mixing preserves some order/information about how the latest distribution was assembled from the beginning
  - Chaotic mixing “forgets” all of this info
- Distinction is valid in time-dependent potential too!



Henriksen & Widrow 1997

Merger relaxation:



(Disclaimer: this is pretty idealized, but this gives an idea of what relaxation would look like based on the interactions of two galaxies which is thought to be more common since elliptical galaxies don’t seem to like being alone)

Limitations: gas and stars are different, who knew?

- Splitting up the process into two parts produces a different result than doing one process that’s the same length
- Result is by nature unpredictable, galaxies are pretty complex
- More than one mechanism necessary to explain evolution
- Ellipticals don’t entirely randomize: some still maintain some slow rotation, so they’ve still got information about how they were assembled

Sources

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