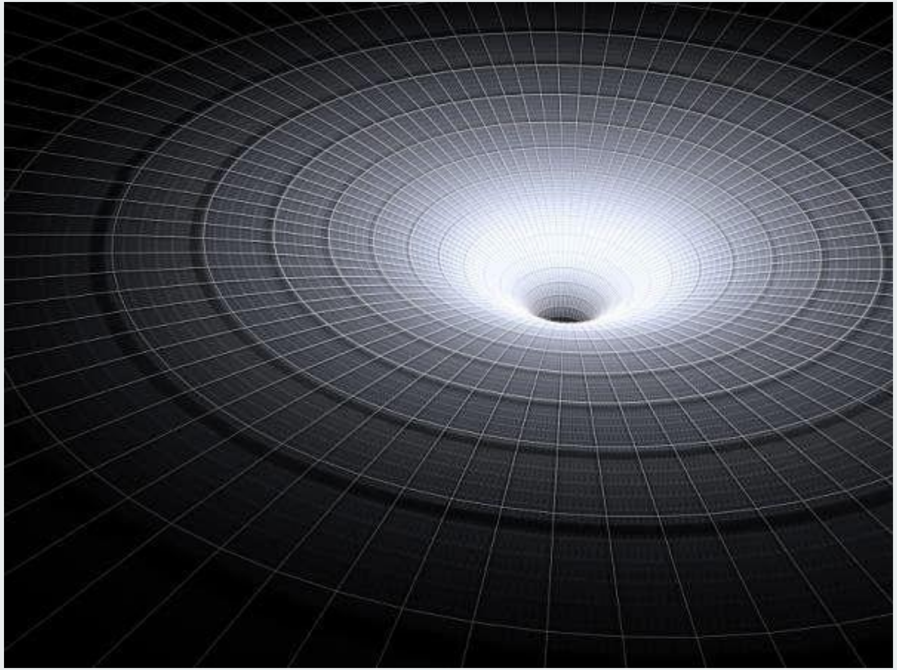


Entropy and Horizons: The Thermodynamics of Spacetime

By: David Zegeye

Motivation

- Gravity is an anomaly in physics
- Entropy of Black Hole is characterized by its geometry..why?
- GR as an Equation of State for gravity

Assumptions

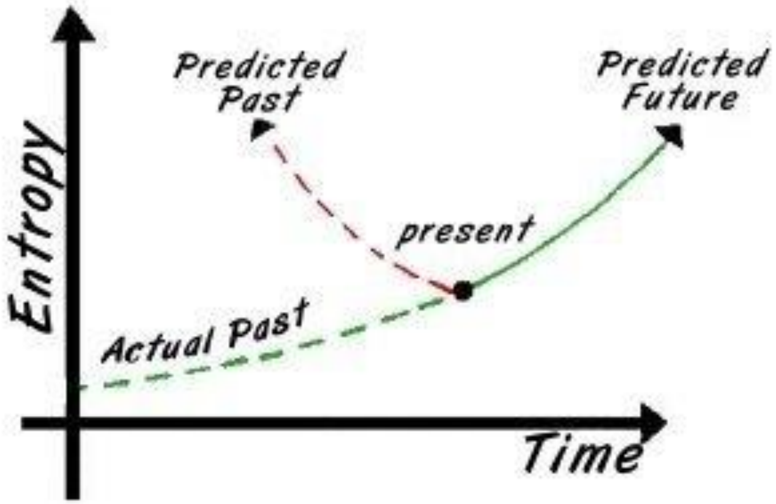
- Thermodynamic Limit (i.e. ignore quantum mechanics)
- Dropping higher order terms
- 1st Law/Conservation of Energy holds true

What is a Causal Horizon?

Geometric view: One-way flow of information in a spacetime boundary

Ex: Black Hole horizon, Expansion of Universe, can’t travel faster than light

Close hints and connections to Thermodynamics (2nd Law of Thermo)



Source: https://xbanguyen.com/category/physics/

Heat and Energy

$$\delta Q = T ds$$

$$T = \frac{\hbar\kappa}{2\pi}; ds = \eta\delta A$$

$$\delta Q = T ds = \frac{\hbar\kappa}{2\pi}\eta\delta A$$

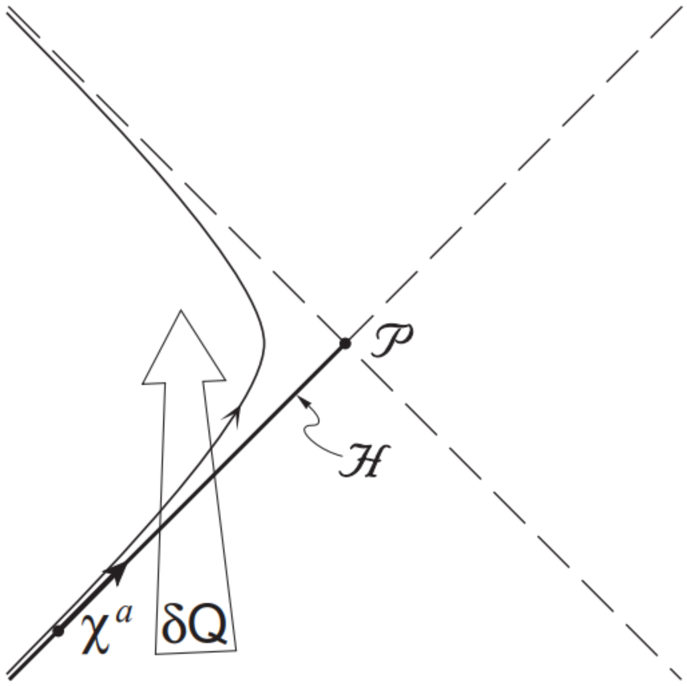
- Heat = energy flow for a horizon
- Let's express temperature in terms of energy flux across a horizon and entropy in terms of the geometry of the horizon

$$\delta Q = T ds$$

$$\delta Q = \int_{\mathcal{H}} T_{ab} \chi^a dV^a$$

$$\chi^a = \kappa \lambda k^a ; dV^a = k^a d\lambda dA$$

- Generalized heat flow in terms of geometry of the system
- $T_{ab} = \mathbf{\Sigma_{a,b}} T_{ab}$, where a & b are dimensions of the system
- T_{ab} (Stress-Energy Tensor) can be thought of the total energy in the system



Source: Jacobson (1995)

$$\delta A = -\int_{\mathcal{H}} \theta d\lambda dA$$

$$\theta \approx -\lambda R_{ab} k^a k^b$$

- Path dependent area of our horizon
- Expansion coefficient of horizon
- R_{ab} is the Ricci Tensor. Describes the path of an object in geometry

$$\delta Q = \frac{\hbar\kappa}{2\pi}\eta\delta A$$

$$-\kappa\int_{\mathcal{H}} T_{ab}\lambda k^a k^b d\lambda dA = -(\frac{\hbar\kappa}{2\pi})\eta\int_{\mathcal{H}} \lambda R_{ab}k^a k^b d\lambda dA$$

$$T_{ab} = (\frac{\hbar\eta}{2\pi})(R_{ab} + f g_{ab})$$

General Relativity Appears!

$$R_{ab} + \frac{1}{2} R g_{ab} = \frac{2\pi}{\hbar \eta} T_{ab}$$

Significance

A unification between GR and Thermodynamics (which one is more fundamental?)

Analogy of GR gravity to sound waves being an emergent property (i.e. cannot be quantized, despite being emergent from a quantum mechanical system)

Indicates GR is a local equilibrium limit of gravity. A “non-equilibrium” limit is necessary for quantized spacetime

Sources

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