Week 12

W-1

Warmups

Problem 1

Schroeder Prob 5.55

Begin with Clausius-Clapeyron ... goal is to show that assuming L const and $\Delta V = V_{gas}$ via ideal gas law, that

$$P = \text{const}\ e^{-L/RT} \quad \text{the vapour pressure equ.}$$

Note: const above must be $P(T=\infty)$

Soln: We will assume $V_{gas} = \frac{RT}{P}$. We then are solving $\frac{dP}{dT} = \frac{L}{T V_{gas}} = \frac{LP}{RT^2}$

$$\Rightarrow \frac{dP}{P} = \frac{L}{R}\frac{dT}{T^2}$$

so integrate to get

$$\ln P = -\frac{L}{RT} + \underbrace{C}_{const} \Rightarrow \underline{\underline{P = \overbrace{e^{C}}^{\text{another const.}}\, e^{-L/RT}}}$$

Warmup Problem 2 **Schroeder 7.66** (Bose-Einstein Condensate ... some numbers

W-2

Part a

For an ^{87}Rb atom in a cube-shaped box of width 10^{-5}m, using eq. 7.118, the g.s. energy is

$$\begin{aligned}
\varepsilon_0 &= \frac{h^2}{8mL^2}(1^2+1^2+1^2) \\
&= \frac{3}{8}\frac{(6.63\times10^{-34})^2}{(87)(1.66\times10^{-27})(10^{-5})^2} \\
&= 1.14\times10^{-32}J \\
&= 7.1\times10^{-14}eV
\end{aligned}$$

Part b

Using eq. 126, the condensation temperature is

$$kT_c = (0.527)\left(\frac{h^2}{2\pi mL^2}\right)N^{2/3} = (0.224)N^{2/3}\varepsilon_0$$

If we have 10,000 atoms in the box, kT_c is larger than ε_0 by a factor of approximately 100, so $T_c = 8.6\times10^{-8}$K.

Part c

At $T = 0.9T_c$, the number of atoms in the ground state is

$$N_0 = N\left(1-\left(\frac{T}{T_c}\right)^{3/2}\right) = N(1-0.9^{3/2}) = 0.146N$$

This is 1460 for $N = 10000$. So, by eq. 7.120

$$\varepsilon_0 - \mu = \frac{kT}{N_0} = \frac{(0.9)(7.4\times10^{-12})}{1460} = 4.6\times10^{-15}$$

$$\Rightarrow \mu \approx 1.05\times10^{-32}\ J$$

So the chemical potential is below the ground state energy by $0.065\varepsilon_0$. The first excited state energy is

$$\varepsilon_1 = \frac{h^2}{8mL^2}(2^2+1^2+1^2) = \frac{6h^2}{8mL^2} = 2\varepsilon_0$$

So we have the expected number of particles as

$$N_1 = \frac{1}{e^{(\varepsilon_1-\mu)/kT}-1} = \frac{1}{e^{1.065\varepsilon_0/kT}-1} = \frac{1}{e^{1.065/(0.9)(7.4\times10^{-12})}} = 87$$

So there are 87 atoms in each of the 3 degenerate first excited states, for total of 261.