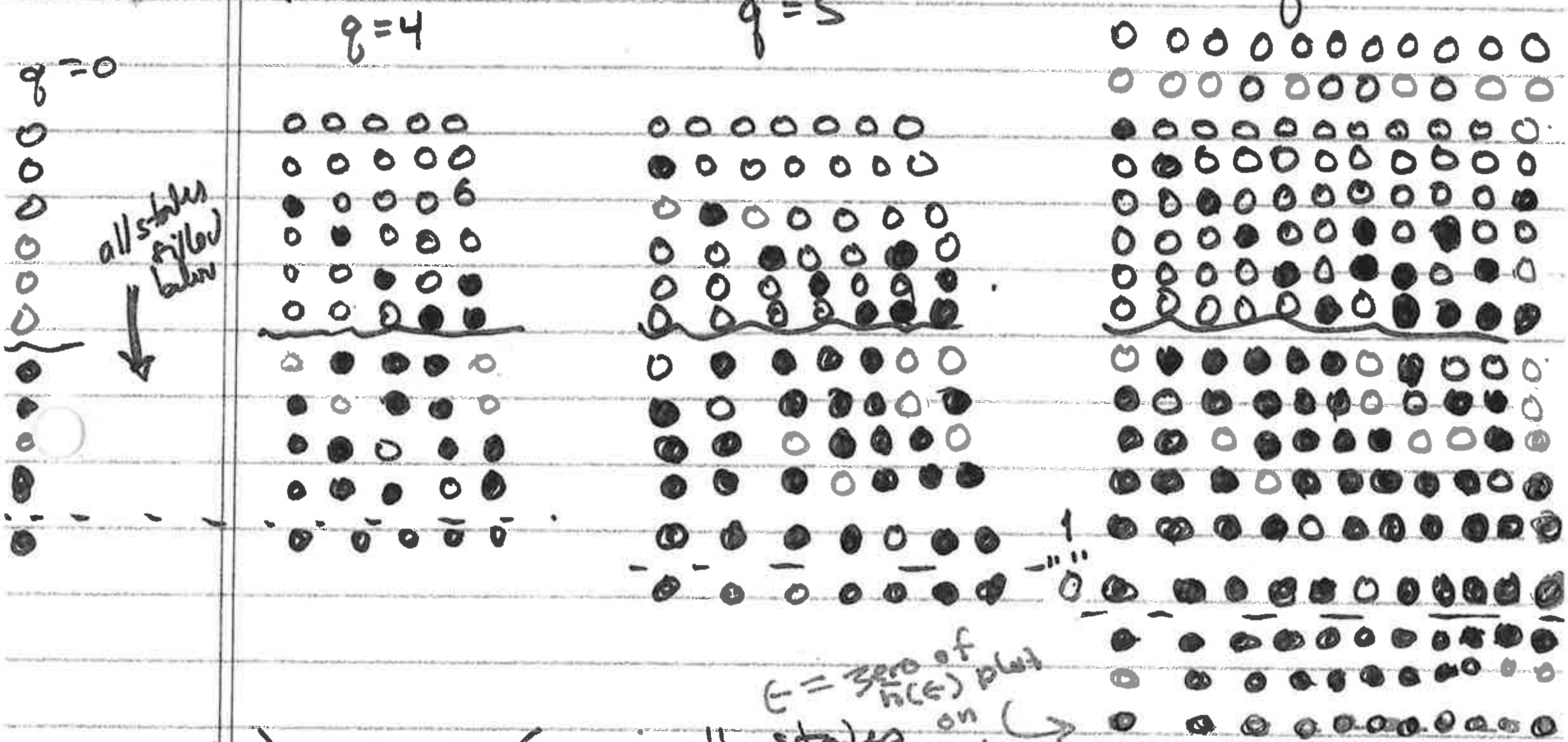
Seminar 11

Warmup Problems

Warmup Problem 2

Counting Fermions
Schroeder 7.16

(a) Draw "dot diagram" for $q = 4, 5, 6$ (energy $4\eta, 5\eta, 6\eta$) with N identical fermions ...



ϵ = zero of $\bar{n}(\epsilon)$ plot on next page

(b) $q = 6$; 11 states

level	prob		level (cont)	prob
0	10	/11	10	1/11
1	10		11	1/11
2	9		12	0
3	8		13	0
4	7		14	0
5	6		⋮	⋮
6	5			
7	4			
8	3			
9	2			

← This is called $\epsilon = \mu$ for plot on Next page (beside level 0)

below "q" all states are occupied ⇒ prob = 11/11 for q = 6

W2-2

FermiList is $\{\epsilon, \bar{n}(\epsilon)\}$

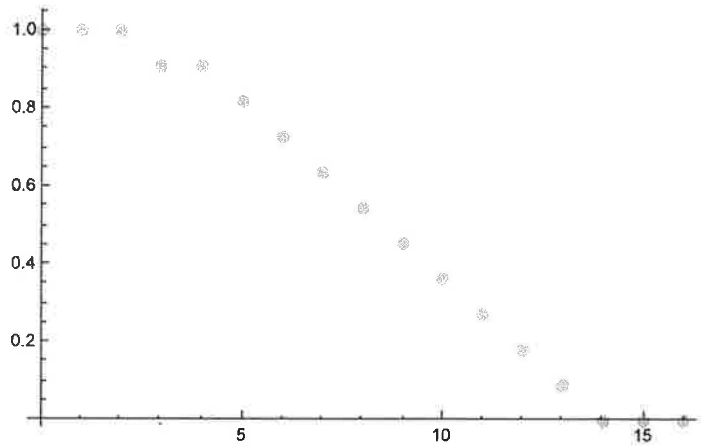
Here are occupancy numbers vs. energy for q = 6. We start with a few levels where all states are occupied so it looks more like a Fermi funciton! We similarly end with a bunch of unoccupied levels.

```
FermiList = {{0, 1}, {1, 1}, {2, 1}, {3, 0.90909}, {4, 0.90909}, {5, 0.818181},
   {6, 0.727272}, {7, .636363}, {8, 0.545454}, {9, 0.454545}, {10, 0.36363},
   {11, 0.272727}, {12, 0.181818}, {13, 0.090909}, {14, 0}, {15, 0}, {16, 0}}
```

```
{{0, 1}, {1, 1}, {2, 1}, {3, 0.90909}, {4, 0.90909}, {5, 0.818181},
 {6, 0.727272}, {7, 0.636363}, {8, 0.545454}, {9, 0.454545}, {10, 0.36363},
 {11, 0.272727}, {12, 0.181818}, {13, 0.090909}, {14, 0}, {15, 0}, {16, 0}}
```

see entries of List for $\epsilon = 8$ & $\epsilon = 9$

```
FermiPlot = ListPlot[FermiList]
```



(c) What is μ? It is where $\bar{n}(\epsilon) \approx 1/2$ so I take it as $\mu = 8.5$

Must keep in mind, though my $\epsilon = 0$ is arbitrary.

```
ClearAll[x, T]

FindFit[Evaluate[FermiList], 1 / (Exp[(x - 8.5) / T] + 1), {T}, x]
```

```
{T → 2.13806}
```

$T \approx 2.14\, \eta/k$

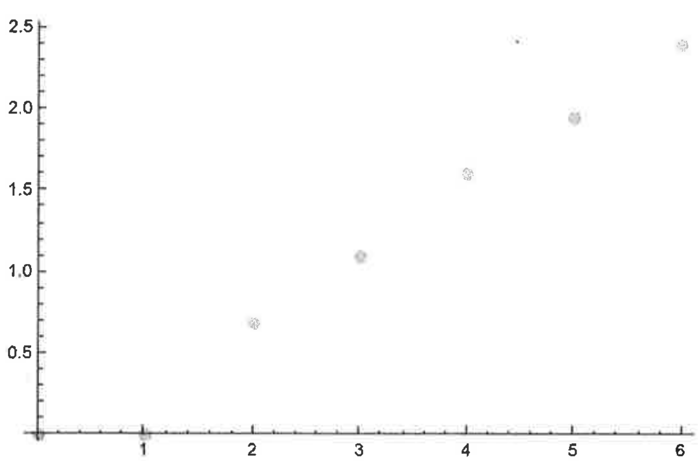
(d)

```
EntropyList = {{0, 0}, {1, 0}, {2, 0.69}, {
   3, 1.10}, {4, 1.61}, {5, 1.95}, {6, 2.40}}
```

```
{{0, 0}, {1, 0}, {2, 0.69}, {3, 1.1}, {4, 1.61}, {5, 1.95}, {6, 2.4}}
```

See next page for data going into EntropyList

```
ListPlot[EntropyList]
```



W2-3

```
FindFit[Evaluate[EntropyList], a*x+b, {a, b}, x]
{a → 0.429286, b → -0.180714}
Temperature = 1/0.429286
2.32945
```

$\frac{dS}{dE} = \frac{1}{T} \Rightarrow T = 2.33\ \eta/k$

Not bad agreement with part (b)

(d) (cont)

q	Ω	$S/k = \ln(\Omega)$
0	1	0
1	1	0
2	2	0.69
3	3	1.10
4	5	1.61
5	7	1.95
6	11	2.40

from $q = 0, 1, 2, 3$ done in text