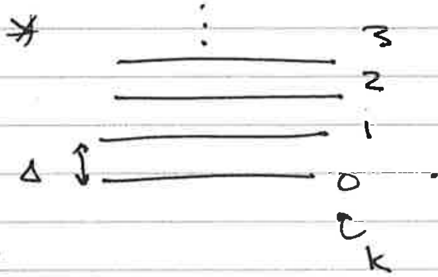
Warmup Prob 1 p. 1

Seminar 10 Warmups

Problem 1, Schroeder 7.10 Counting states

We have 5 particles among nondegen evenly spaced energy levels. Particles could be Bosons - identical
Fermions - identical (spinless) *
Distinguishable



$E_k = k \cdot \Delta$

(a) Ground state?

B	F	D
all in $k=0$ $n_0 = 5$	one particle each in $k = 0,1,2,3,4$	all in $k=0$ $n_0 = 5$ $n_i = 1 \quad i = 0-4$ $n_i = 0 \quad i > 4$
$E_0 = 0$	$E_0 = 10\Delta$	$E_0 = 0$

* NB with spins
two in $k=0$: $n_0 = 2$
two in $k=1$ $n_1 = 2$
one in $k=2$ $n_2 = 1$

(b) if $E = E_0 + \Delta$, allowed states are

B	F	D
four in $k=0$ one in $k=1$ $E_1 = \Delta$ $\Omega = 1$	$n_i = 1 \quad i = 0-3$ $n_4 = 0$ $n_5 = 1$ $n_i = 0 \quad i > 5$ $E_1 = 11\Delta$ $\Omega = 1$	four in $k=0$ one in $k=1$ $E_1 = \Delta$ $\Omega = \binom{5}{1} = 5$

(c) There are two ways to add one more unit of energy for B or D

$n_0 = 3$ or $n_0 = 4$
$n_1 = 2$ $\quad n_1 = 0$
$\qquad\quad n_2 = 1$

These have different multiplicities.

B

$n_0 = 3$	$n_0 = 4$
$n_1 = 2$	$n_1 = 0$
	$n_2 = 1$
$E_2 = 2\Delta$	$E_2 = 2\Delta$
$\Omega = 1$	$\Omega = 1$

Total

$\Omega = 2$ for $E_2 = 2\Delta$

D

$n_0 = 3$	$n_0 = 4$
$n_1 = 2$	$n_1 = 0$
	$n_2 = 1$
$\Omega = \binom{5}{2} = 10$	$\Omega = \binom{5}{1} = 5$

Total

$\Omega = 15$ for $E_2 = 2\Delta$

F

two ways also

$n_0 = 1$	$n_0 = 1$
$n_1 = 1$	$n_1 = 1$
$n_2 = 1$	$n_2 = 1$
$n_3 = 1$	$n_3 = 0$
$n_4 = 0$	$n_4 = 1$
$n_5 = 0$	$n_5 = 1$
$n_6 = 1$	

for $E_2 = 12\Delta$

$\Omega = 2$

Yet another unit of energy? Three ways for each

B (or / or)

n_0	2	3	4
n_1	3	1	0
n_2	0	1	0
n_3	0	0	1
n_4	0	0	0
n_5	0	0	0
n_6	0	0	0
n_7	0	0	0

$\Omega = 3$ for $E = 3\Delta$

F

n_0	1	1	1
n_1	1	1	1
n_2	1	1	0
n_3	1	0	1
n_4	0	0	1
n_5	0	0	1
n_6	0	1	0
n_7	1	0	0

$\Omega = 3$
$E = 13\Delta$

D

n_0	2	3	4
n_1	3	1	0
n_2	0	1	0
n_3	0	0	1
n_4	0	0	0
n_5	0	0	0
n_6	0	0	0
n_7	0	0	0

$E = 3\Delta$

$$\Omega = \binom{5}{2} + \binom{5}{3}\cdot 2 + \binom{5}{4} = 10 + 20 + 5 = 35$$

(d) Probability of seeing E goes like
$$g(E)\, e^{-\beta E} .$$

So consider B & D. It is relatively more likely to see a higher E for D than B, b/c there are so many more ways, Ω for D particles ...

		B	D
eg.	$\dfrac{\Omega_{E=3}}{\Omega_{E=0}}$ =	$\dfrac{3}{1}$	$\dfrac{35}{1}$

Now, though not asked, issue with Fermions is just that gs has higher energy as levels must fill up with 1 particle each. Multiplicity of $E_0 + \Delta$ for F is same as multiplicity of Δ for B where $E_0 = \sum_{i=1}^{N} i \cdot \Delta$

Warmup
Prob 2
P. 1

Problem 2

Schroeder 7.37

We are asked to show

That the Planck function giving spectrum in terms of energy:

$$u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{e^{\beta\epsilon}-1} \quad \text{peaks at } \beta\epsilon = 2.82$$

We could write ϵ in terms of $x = \beta\epsilon$

$\Rightarrow$ find peak of $$\frac{8\pi}{(\beta hc)^3}\frac{x^3}{e^x-1} \equiv f(x)$$

Differentiate: $f'(x) = 0$

$$\Rightarrow \quad 0 = \frac{3x^2}{e^x-1} - \frac{x^3 e^x}{(e^x-1)^2}$$

$$\Rightarrow \quad 3x^2(e^x-1) = x^3 e^x$$

if $x \neq 0$ $$\Rightarrow \quad 3(e^x-1) = x e^x$$

$$\Rightarrow 3e^{-x} = 3 - x$$

$$e^{-x} = 1 - \frac{x}{3}$$

Solve: $$x = 2.82\ldots$$

or Guess + check

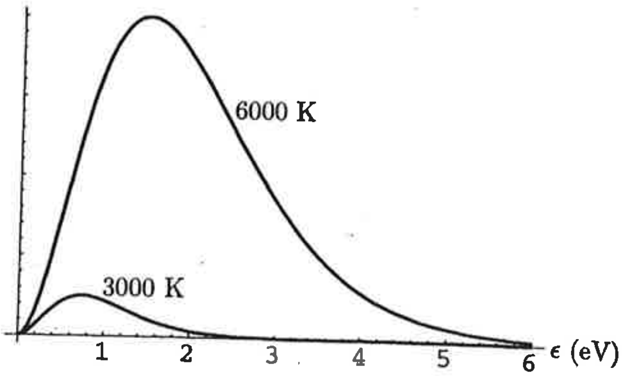
Schroeder

Warmup
Prob 2
p. 2

Problem 7.38. At $T = 3000$ K, $kT = 0.26$ eV, while at $T = 6000$ K, $kT = 0.52$ eV. To plot the Planck spectrum vs. photon energy at each of these temperatures, I used the *Mathematica* instruction

```
Plot[{e^3/(Exp[e/.26]-1),e^3/(Exp[e/.52]-1)},{e,0,6}]
```

in which I've ignored the overall constant in equation 7.84 since the vertical axis of the graph is so tricky to interpret anyway. Here's the resulting plot:



Note that doubling the temperature shifts the peak in the spectrum to the right, to a photon energy exactly twice as large. Much more dramatic, though, is the height of the spectrum: Doubling the temperature increases the total area under the graph by a factor of $2^4 = 16$, as predicted by equation 7.85 or 7.86.