Phys 114

# Seminar 9
## Warmup Problems

2018

W1
W-1

Problem 1) G & T Prob 5.3

(a) $M = N\mu \tanh(\beta\mu B)$

$C = kN(\beta\mu B)^2 \operatorname{sech}^2(\beta\mu B)$

Plot these vs. T & comment on broad max of C at intermediate temperatures...

see Mathematica, next page

(b) Plot $\chi = N\mu^2\beta \operatorname{sech}^2(\beta\mu B)$ vs. T and comment on low & high T behaviors.

(c) Calculate S and discuss its low ($kT \ll \mu B$) and high ($kT \gg \mu B$) temperature behaviors. Does S depend on kT & $\mu B$ separately?

S can be found from $F = E - TS$

$\Rightarrow S = (E-F)/T = -\frac{1}{T}\frac{\partial}{\partial\beta}\ln Z + k\ln Z$;

Thus since $Z = (2\cosh\beta\mu B)^N$ we have

$$S = -kN\underbrace{\left(\frac{\mu B}{kT}\right)}_{\equiv \beta\mu B}\tanh\beta\mu B + kN\ln(2\cosh\beta\mu B)$$

So no... S depends on $\beta\mu B$, Not $kT$ & $\mu B$ separately

see Mathematica plot.

$T\to 0$ / $S \to 0$ and $\partial S/\partial T \to 0$ ✓ $T \to \infty$ S goes to constant! $S \to Nk\ln 2$

W-2

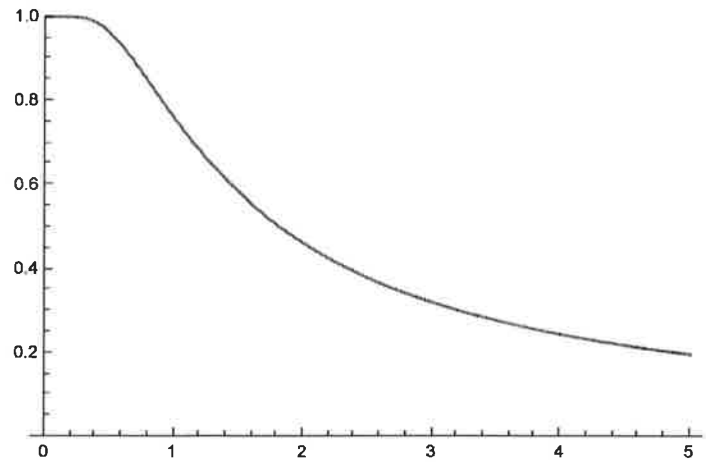
# G&T 5.3

**We will let B = $\mu$ = k = 1 and will plot M and C vs. T. Since these are proportional to N, the number of spins, this extensivity will be implicit:**

In[2]:=
```
MM[T_] := Tanh[1 / T]
```

In[14]:=
```
Plot[MM[T], {T, .001, 5}, PlotRange → {0, 1}]
```

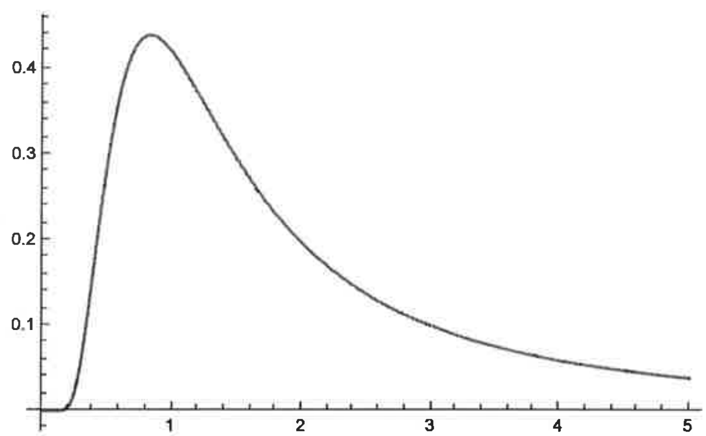
Out[14]=




In[10]:=
```
CC[T_] := (1 / T)^2 * (Sech[1 / T])^2
```

In[15]:=
```
Plot[CC[T], {T, .001, 5}]
```

Out[15]=




**We see that C has a broad maximum, which is in the region of T's where M(T) is falling from the value of 1.0 (perfectly aligned spins) toward 0.0 (randomly aligned spins). In this region, the system is able to take on energy by flipping spins, so it is able to receive energy (from a bath ... ) and this is reflected in a change in temperature.**

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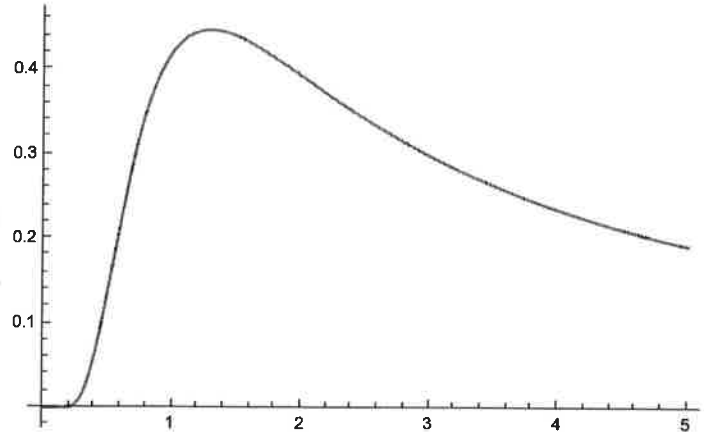
In[18]:=
```
Chi[T_] := (1 / T) * (Sech[1 / T])^2
```

W 1 / W - 3

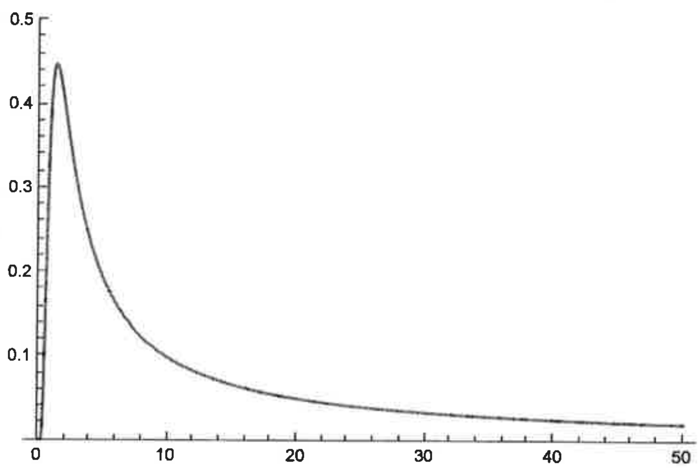
```
In[19]:= Plot[Chi[T], {T, .001, 5}]
```

Out[19]=




```
In[23]:= Plot[Chi[T], {T, .001, 50}, PlotRange → {0, 0.5}]
```

Out[23]=




**We realize that except for different units, and an additional factor of "T", Chi[T] and C[T] have identical forms. So *X* will still go to zero as T->0, and *X* will still go to 0 as T-> ∞ albeit not quite as fast as C will. Both of these are response functions, C is a response to added energy and *X* is a response to added magnetic field strength.**

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```
In[36]:= SS[T_] := - (1 / T) * Tanh[1 / T] + Log[2 * Cosh[1 / T]]

In[44]:= SS[5.0]

Out[44]= 0.67354
```

wr1
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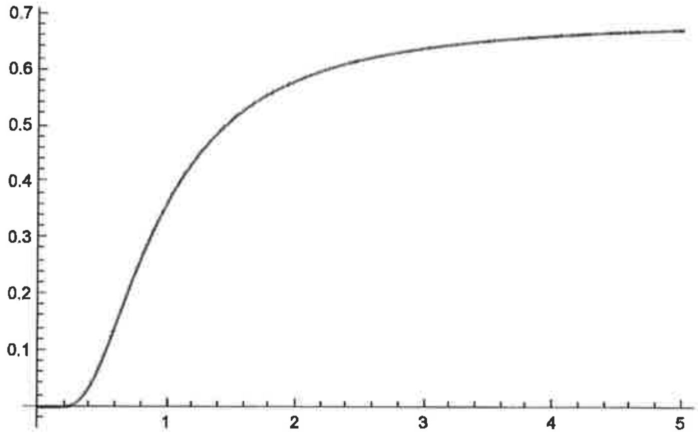
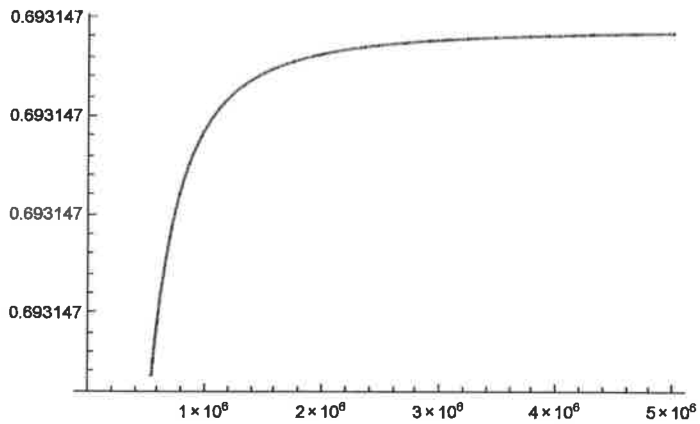
In[47]:= `Plot[SS[T], {T, .01, 5}]`

Out[47]=




In[50]:= `Plot[SS[T], {T, .01, 5000000}]`

Out[50]=



Warmup 2

W2: Microcanon derivn of M
Schroeder 3.19-ish
fill in steps from 3.30 – 3.32 (or 3.33)

Start with

$$\frac{S}{k} \cong N\ln N - n\ln n - (N-n)\ln(N-n)$$

where $n \equiv N_\uparrow$ for ease of writing.

Now

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_N = \frac{\partial n}{\partial U}\frac{\partial S}{\partial n} = -\frac{1}{2\mu B}\frac{\partial S}{\partial n} \quad (3.29)$$

because $U = -\mu B n + \mu B(N-n) = -\mu B(2n - N)$

$$\therefore \frac{\partial n}{\partial U} = \frac{1}{\partial U/\partial n} = \frac{1}{-2\mu B} \quad :)$$

Ok soln begins ....

$$\frac{\partial S}{\partial n} = k\left[-\ln n - 1 + \ln(N-n) + 1\right]$$

$$= k\ln\frac{(N-n)}{n}$$

$$= k\ln\left(\frac{\left(\frac{N}{2} + \frac{U}{2\mu B}\right)}{\left(\frac{N}{2} - \frac{U}{2\mu B}\right)}\right)$$

$\frac{U}{\mu} = N - 2n$
$\therefore n = \frac{N}{2} - \frac{U}{2\mu B}$

$$\therefore \frac{1}{T} = -\frac{k}{2\mu B}\ln\left(\frac{\left(\frac{N}{2} + \frac{U}{2\mu B}\right)}{\left(\frac{N}{2} - \frac{U}{2\mu B}\right)}\right) = +\frac{k}{2\mu B}\ln\left(\frac{\left(N - \frac{U}{\mu B}\right)}{\left(N + \frac{U}{\mu B}\right)}\right)$$

(3.29 ⇒)

which is (3.30)

We thus invert to get

$$\frac{2\mu B}{kT} = \ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right)$$

$$\Rightarrow \left(N + \frac{U}{\mu B}\right) e^{2\mu B/kT} = N - U/\mu B$$

$$\Rightarrow U\left[e^{2\mu B/kT} + 1\right] = N\mu B\left[1 - e^{2\mu B/kT}\right]$$

$$\Rightarrow U = N\mu B \frac{\left(1 - e^{2\mu B/kT}\right)}{\left(1 + e^{2\mu B/kT}\right)} = -N\mu B \tanh\left(\frac{\mu B}{kT}\right)$$

Eq. (3.31)

Since Bulk Magnetizn $M = -U/B$, we have

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right)$$

Eq. (3.32)

Now, find $C_B = \left(\frac{\partial U}{\partial T}\right) = +\frac{N\mu^2 B^2 k}{k^2} \operatorname{sech}^2\left(\frac{\mu B}{kT}\right)$

Eq (3.33)