Seminar 1

Problem Solutions

Phys 114

1. reversible or not? G + T 2.6

(a) Squeezing a plastic bottle: irreversible if you truly deform the plastic, reversible if you do not b/c it can spring back into shape *

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is
reversible
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n system

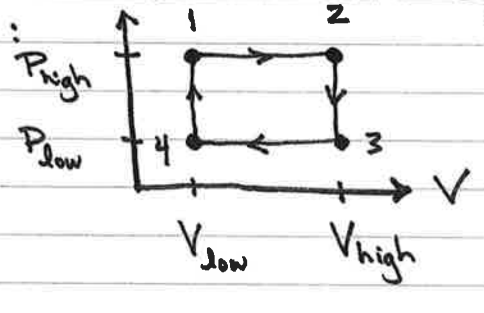
(b) Ice melting in water: irreversible in sense that you can refreeze an equivalent amount of ice, but the specific H_2O molecules that used to make up the ice (say) cube cant be corralled to reform the cube

(c) Movement of a real, frictional piston: irreversible ... work → KE of piston → heat lost due to friction ... jiggling of molecules that make up the object

(d) Pumping air into a tire: reversible if done slowly enough ... you can let the air out again. However, if you consider expansion of tire, real elastic medium, energy will be lost as heat due to * resistive forces... so irreversible.

2. Wok G + T 2.8

Example 2.1 involves the cycle in Fig 2.4 in PV plane:



(a) Because the system was returned to its original pressure and volume, why is the net amount of work done not zero?

Solution. The work done by the system at pressure P_2 is greater in magnitude than the work done on the system at pressure P_1.

(b) What would be the work done if the gas were taken from 1 → 2 → 3 and then back to 1 along the diagonal path connecting 3 and 1?

Solution. The work done on the system from 1 → 3 is $W_{1\to 3} = -P_{\text{high}}(V_{\text{high}} - V_{\text{low}})$. No work is done on the system from 2 → 3. To find the work done on the system along the diagonal from 3 → 1, we write

$$P = -aV + b, \tag{S2.2}$$

where

$$a = \frac{P_{\text{high}} - P_{\text{low}}}{V_{\text{high}} - V_{\text{low}}}. \tag{S2.3}$$

Note that a is defined so that it is positive. We find b by requiring that $P = P_{\text{low}}$ when $V = V_{\text{high}}$:

$$P_{\text{low}} = -aV_{\text{high}} + b, \tag{S2.4a}$$

or

$$b = P_{\text{low}} + aV_{\text{high}}. \tag{S2.4b}$$

To find the work done, we substitute (S2.2) into the relation

$$W_{\text{diag}} = -\int_{V_{\text{high}}}^{V_{\text{low}}} P\,dV = -\int_{V_{\text{high}}}^{V_{\text{low}}} [-aV + b)]\,dV \tag{S2.5a}$$

$$= \frac{a}{2}\left[V_{\text{low}}^2 - V_{\text{high}}^2\right] - b(V_{\text{low}} - V_{\text{high}}) = (V_{\text{low}} - V_{\text{high}})\left[\frac{a}{2}(V_{\text{low}} + V_{\text{high}}) - b\right]. \tag{S2.5b}$$

We first eliminate b [(see (S2.4b)] and obtain

$$W_{\text{diag}} = (V_{\text{low}} - V_{\text{high}})\left[\frac{a}{2}(V_{\text{low}} - V_{\text{high}}) - P_{\text{low}}\right]. \tag{S2.5c}$$

We then eliminate a and find

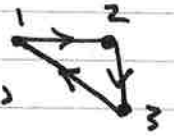
$$W_{\text{diag}} = (V_{\text{low}} - V_{\text{high}})\left[\frac{1}{2}\frac{(P_{\text{high}} - P_{\text{low}})}{(V_{\text{high}} - V_{\text{low}})}(V_{\text{low}} - V_{\text{high}}) - P_{\text{low}}\right] \tag{S2.5d}$$

$$= (V_{\text{low}} - V_{\text{high}})\left[-\frac{1}{2}(P_{\text{high}} - P_{\text{low}}) - P_{\text{low}}\right] \tag{S2.5e}$$

$$= -\frac{1}{2}(V_{\text{low}} - V_{\text{high}})(P_{\text{low}} + P_{\text{high}}). \tag{S2.5f}$$

Let
$\equiv$ high
$l \equiv$ low

Thus, work on the entire path, W_{tot},



is $W_{1\to 3} + W_{diag} = -P_h V_h + P_h V_l +$

$+ \frac{1}{2}P_h V_h - \frac{1}{2}P_h V_l - \frac{1}{2}P_l V_l + \frac{1}{2}P_l V_h$

or

$$W_{tot} = \frac{1}{2}\left[-P_h V_h + P_h V_l - P_l V_l + P_l V_h\right]$$

[illegible] even better, realizing [illegible] W_{tot} is, by Area enclosed by cycle, this is

$$W_{tot} = -\frac{1}{2}\left[(P_h - P_l)(V_h - V_l)\right]$$

$-$ Area of triangle :)

3. adiabatic and isothermal compression: G+T 2.17

Problem 2.17. Air initially at 20°C is compressed by a factor of 15.

(a) What is the final temperature assuming that the compression is adiabatic and $\gamma \approx 1.4$, the value of γ for air in the relevant range of temperatures? By what factor does the pressure increase?

Solution. For a quasistatic adiabatic process and an ideal gas we have that

$$TV^{\gamma-1} = \text{constant}. \tag{S2.10}$$

Hence

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293 \times 15^{0.4} = 293 \times 2.95 = 865.6\,\text{K}. \tag{S2.11}$$

From $PV^\gamma = \text{constant}$ we have

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = 15^{1.4} = 15 \times 2.95 = 44.25. \tag{S2.12}$$

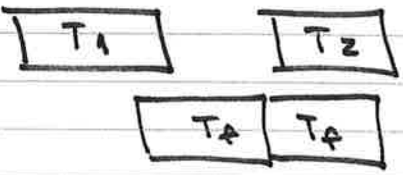
(b) By what factor does the pressure increase if the compression is isothermal?

Solution. Because $P \propto V$ at constant T, the pressure increases by a factor of 15.

(c) For which process does the pressure change more?

Solution. In this case the pressure increases by almost a factor of 3 more for the adiabatic process.

4. Transfer of heat between two bodies: B+B 2.5



$$Q_1 + Q_2 = 0$$

$$\Rightarrow \quad C_1T_1 + C_2T_2 = C_1T_f + C_2T_f$$

$$\therefore \quad T_f = \frac{C_1T_1 + C_2T_2}{C_1 + C_2}$$

a weighted average of the initial temperatures

Let's suppose $C_1 \gg C_2$. Then

$$T_f = \frac{T_1 + \frac{C_2}{C_1}T_2}{1 + C_2/C_1} \approx \left(T_1 + \left(\frac{C_2}{C_1}\right)T_2\right)\left(1 - \frac{C_2}{C_1}\right)$$

$$\approx T_1 + \frac{C_2}{C_1}(T_2 - T_1)$$

QED ☺

5. Isothermal expansion: B+B 11.1

For ideal gas, E cannot change if T does not. That is, $\Delta E \equiv 0$ or

$$\left(\frac{\partial U}{\partial V}\right)_T \equiv 0 .$$ Thus

work on gas:
$$W_{on} = -\int_{V_1}^{V_2} p\,dV = -\int_{V_1}^{V_2} \frac{RT_0}{V}\,dV = -RT_0 \ln\left(\frac{V_2}{V_1}\right)$$

The work done by the gas is the negative of this: $W_{by} = RT_0 \ln\left(\frac{V_2}{V_1}\right) > 0$ ✓

The heat flow into the gas is, since $\Delta E = 0 = W_{on} + Q_{into}$

$$\Rightarrow Q_{into} = -W_{on} = W_{by} = RT_0 \ln\left(\frac{V_2}{V_1}\right)$$

$$dx(r,y) = \left(\frac{\partial x}{\partial r}\right)_y dr + \left(\frac{\partial x}{\partial y}\right)_r dy$$

$$\therefore \left(\frac{\partial x}{\partial r}\right)_\theta = \left(\frac{\partial x}{\partial r}\right)_y \underset{\overset{\downarrow}{1}}{\left(\frac{\partial r}{\partial r}\right)_\theta} + \left(\frac{\partial x}{\partial y}\right)_r \left(\frac{\partial y}{\partial r}\right)_\theta$$

Now $\left(\frac{\partial x}{\partial r}\right)_y = \frac{\partial}{\partial r}\sqrt{r^2-y^2}\Big|_y$

$$= \frac{r}{\sqrt{r^2-y^2}} = \frac{r}{x} = \frac{1}{\cos\theta}$$

And similarly $\left(\frac{\partial x}{\partial y}\right)_r = \frac{\partial}{\partial y}\sqrt{r^2-y^2}\Big|_r$

$$= \frac{-y}{x} = \frac{-\sin\theta}{\cos\theta}$$

while $\left(\frac{\partial y}{\partial r}\right)_\theta = \frac{\partial}{\partial r} r\sin\theta\Big|_\theta = \sin\theta$

So finally

$$\left(\frac{\partial x}{\partial r}\right)_\theta = \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta} = \frac{1}{\cos\theta}(1-\sin^2\theta)$$

$$= \frac{\cos^2\theta}{\cos\theta} = \cos\theta$$ ☺ right!

6 B+B 11.4

In polar coords, $x = r\cos\theta$ and $y = r\sin\theta$ which seems to imply $\frac{\partial x}{\partial r} = \cos\theta = \frac{x}{r}$. But $x^2 + y^2 = r^2$ so $2x\frac{\partial x}{\partial r} = 2r \Rightarrow \frac{\partial x}{\partial r} = r/x$. Thus $\frac{\partial x}{\partial r} = \frac{\partial r}{\partial x}$? Wazup?

The problem here is that the question has been (deliberately) misleading about writing down which variables are held constant. One can think of x as a function of r and θ, i.e. $x = x(r, \theta)$, so from the equation

$$x = r\cos\theta,$$

it follows that

$$\left(\frac{\partial x}{\partial r}\right)_\theta = \cos\theta = \frac{x}{r}.$$

One can also think of $x = x(y, r)$ from the equation

$$x^2 = r^2 - y^2,$$

in which case

$$2x\left(\frac{\partial x}{\partial r}\right)_y = 2r \qquad \Longrightarrow \qquad \left(\frac{\partial x}{\partial r}\right)_y = \frac{r}{x}.$$

Hence what is actually true is that

$$\left(\frac{\partial x}{\partial r}\right)_\theta = \left(\frac{\partial r}{\partial x}\right)_y.$$

Moral of the story: Think carefully about what is being held constant in a partial derivative.

More to point... see 6a

7: Partial deriv's for a Thermal process:

B+B 12.3 Explain why we can write $\text{đ}Q = C_p\,dT + A\,dp$ & $\text{đ}Q = C_v\,dT + B\,dV$ subtract to show $(C_p - C_v)\,dT = B\,dV - A\,dp$ so T constant $\Rightarrow \left(\frac{\partial p}{\partial V}\right)_T = \frac{B}{A}$

In adiabatic change, show $dp = -\left(\frac{C_p}{A}\right)dT$; $dV = -\left(\frac{C_v}{B}\right)dT$

Hence show a, b and c below

When T is constant, $\mathrm{d}T = 0$ and hence

see 6b

$$\left(\frac{\partial p}{\partial V}\right)_T = \frac{B}{A}.$$

Now if $\text{đ}Q = 0$, the first two equations immediately yield

$$\begin{aligned} \mathrm{d}p &= -(C_p/A)\mathrm{d}T, \\ \mathrm{d}V &= -(C_V/B)\mathrm{d}T. \end{aligned}$$

Hence

$$\left(\frac{\partial p}{\partial V}\right)_{\text{adiabatic}} = \gamma\left(\frac{\partial p}{\partial V}\right)_T. \qquad a$$

If p is constant, then we have $C_p - C_V = B\left(\frac{\partial V}{\partial T}\right)_p$ and hence use of $\mathrm{d}V = -(C_V/B)\mathrm{d}T$ yields

$$\left(\frac{\partial V}{\partial T}\right)_{\text{adiabatic}} = -\frac{C_V}{B} = \frac{1}{1-\gamma}\left(\frac{\partial V}{\partial T}\right)_p. \qquad b$$

If V is constant, then we have $C_p - C_V = -A\left(\frac{\partial p}{\partial T}\right)_V$ and hence use of $\mathrm{d}p = -(C_p/A)\mathrm{d}T$ yields

$$\left(\frac{\partial p}{\partial T}\right)_{\text{adiabatic}} = -\frac{C_p}{A} = \frac{\gamma}{\gamma - 1}\left(\frac{\partial p}{\partial T}\right)_V. \qquad c$$

Note

A & B are not constant like "R" but are rather calculable from Eq. of state + involve T, P, V, N For example:

$$\text{đ}Q = dU \underbrace{- \text{đ}W}_{\text{spose this is reversible, pressure work + is } p\,dV = -\text{đ}W}$$

Still $dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$

$U(T,V)$ say ... (N implicit).

Then Since $C_V = \left(\frac{\text{đ}Q}{\partial T}\right)_V$

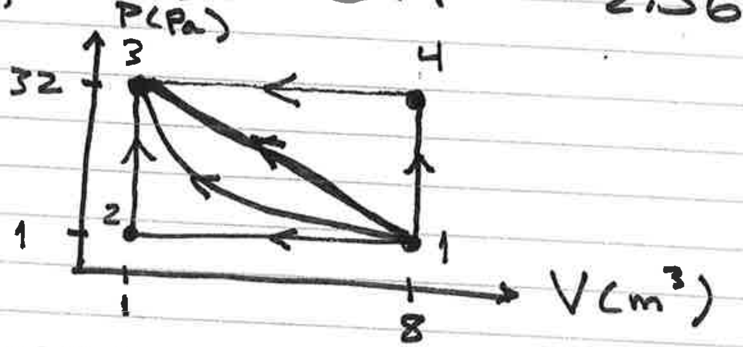
pressure work done $= \left(\frac{\partial U}{\partial T}\right)_V$

$$\text{đ}Q = C_V\,dT + \left(\frac{\partial U}{\partial V}\right)_T dV + p\,dV$$

$$= C_V\,dT + \underbrace{\left(\left(\frac{\partial U}{\partial V}\right)_T + p\right)}_{\text{The "constant" } B.} dV$$

In case of ideal gas, $B \equiv p$!

8: Work, heat, & thermal energy for different thermo processes: G+T 2.56



This is monatomic ideal gas
$\Rightarrow C_V = \frac{3}{2}R$
$C_P = \frac{5}{2}R$
$\gamma = 5/3$

Problem 2.56. Consider the various processes shown in Figure 2.17 and calculate W, the total work done on the system and Q, the total energy absorbed by heating the system. Assume that the system is an ideal gas.

(a) The volume is changed quasistatically along the curved path $1 \to 3$ while the gas is kept thermally isolated.

Solution. $Q = 0$ because the process is adiabatic. The work done is

$$W = \frac{P_3V_3 - P_1V_1}{\gamma - 1}. \tag{S2.69}$$

Hence

$$W = \frac{(32 \times 1) - (1 \times 8)}{5/3 - 1} = \frac{24}{2/3} = 36\,\text{J}. \tag{S2.70}$$

Because $Q = 0$, $\Delta E = 36\,\text{J}$.

(b) The system is compressed from its original volume of $V_1 = 8\,\text{m}^3$ to its final volume $V_3 = 1\,\text{m}^3$ along the path $1 \to 2$ and $2 \to 3$. The pressure is kept constant at $P_1 = 1\,\text{Pa}$ and the system is cooled to maintain constant pressure. The volume is then kept constant and the system is heated to increase the pressure to $P_2 = 32\,\text{Pa}$.

Solution. Work is done from $1 \to 2$, $W = -1(1 - 8) = 7\,\text{J}$. No work is done from $2 \to 3$. Because ΔE is independent of the path, $Q = \Delta E - W = 36 - 7 = 29\,\text{J}$.

(c) The pressure is changed at constant volume from $1 \to 4$ and then at constant pressure from $4 \to 3$. The two steps of the preceding process are performed in opposite order.

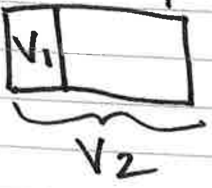
Solution. For $1 \to 4$ no work is done. For $4 \to 3$, $W = -32(1 - 8) = 32 \times 7 = 224\,\text{J}$. Hence $Q = 36 - 224 = -188\,\text{J}$.

(d) Diagonal path from $1 \to 3$. The volume is decreased and the system is heated so that the pressure is proportional to the volume.

Solution. $W = [\frac{1}{2}(32 - 1) \times 7]$ (area of triangle) $+ 7$ (area of rectangle) $= 115.5\,\text{J}$. and $Q = -79.50$

so 79.5 J is taken away from system

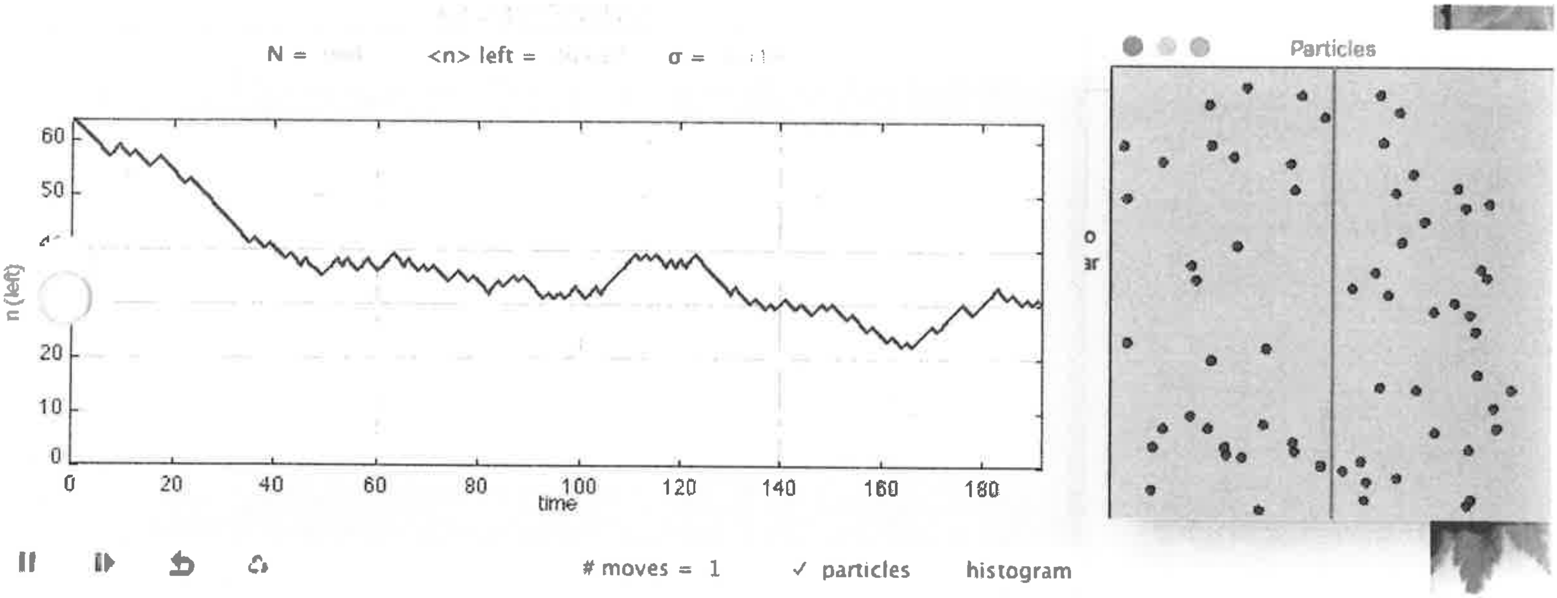
9: Free expansion of ideal gas from $V_1 \to V_2$

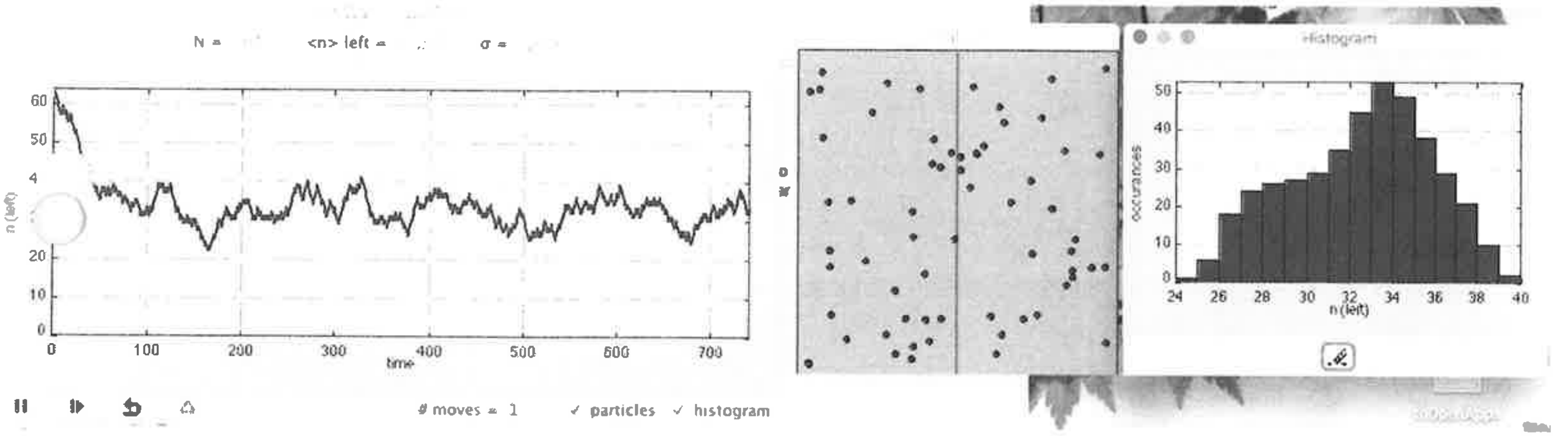


Adiabatic ... $Q = 0$

and $W = 0$

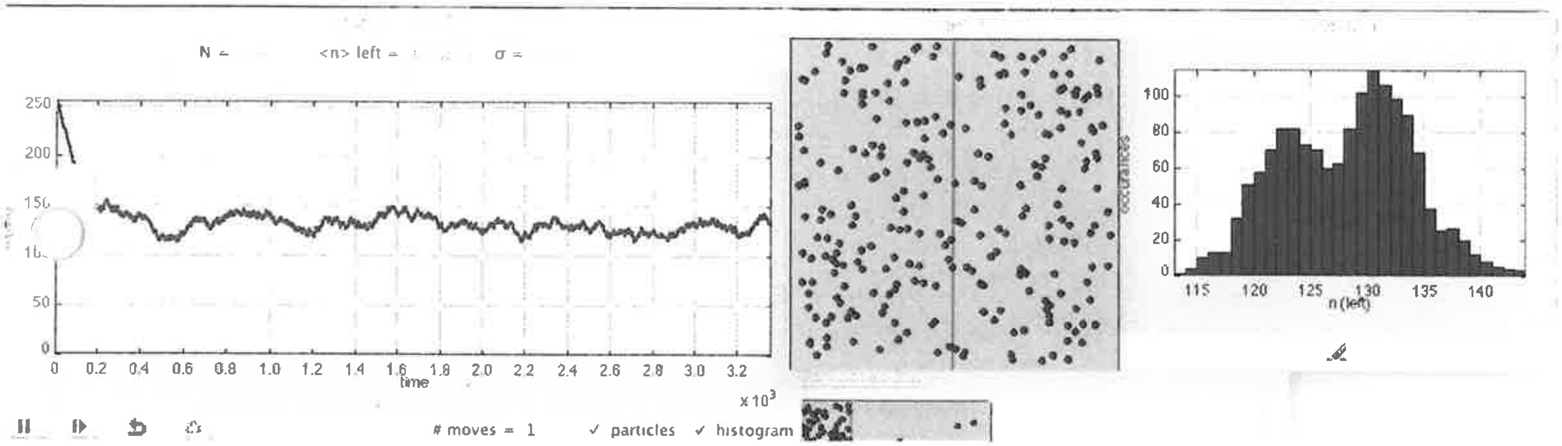
$$\Rightarrow \underline{\Delta E = 0} \Rightarrow \underline{\Delta T = 0}$$


N =
<n> left =
σ =
Particles
n (left)
60
50
20
10
0
time
0
20
40
60
80
100
120
140
160
180
moves = 1
✓ particles
histogram


N = <n> left = σ =

moves = 1

✓ particles ✓ histogram



N =

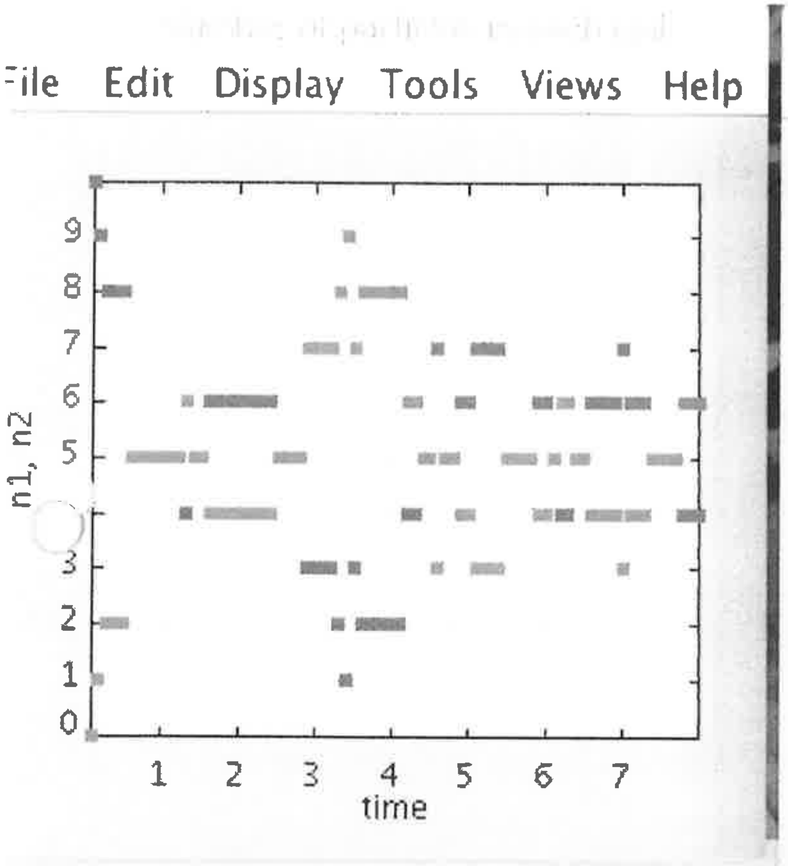
<n> left =

σ =

moves = 1

✓ particles

✓ histogram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n1, n2
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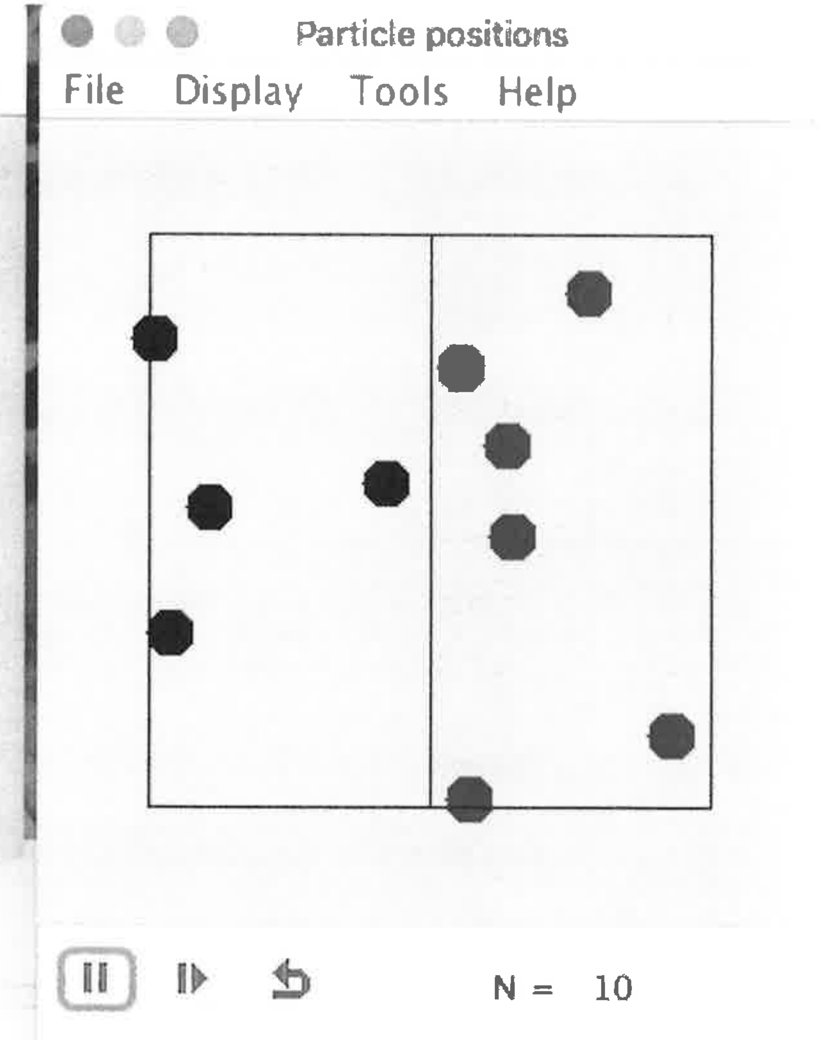
Particle positions
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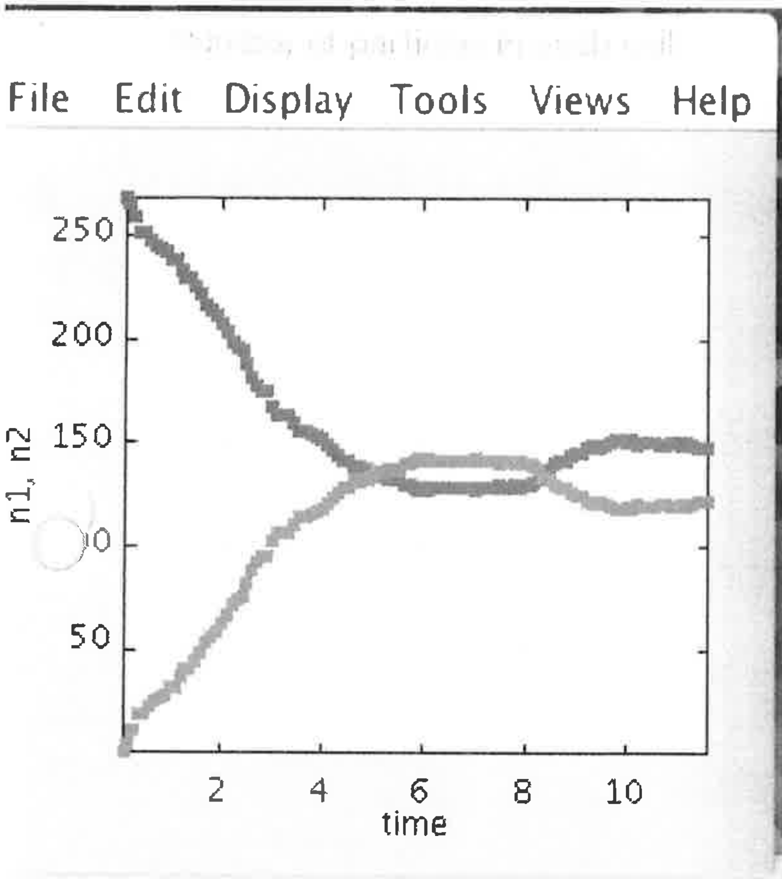
N = 10

20


Number of particles in each cell
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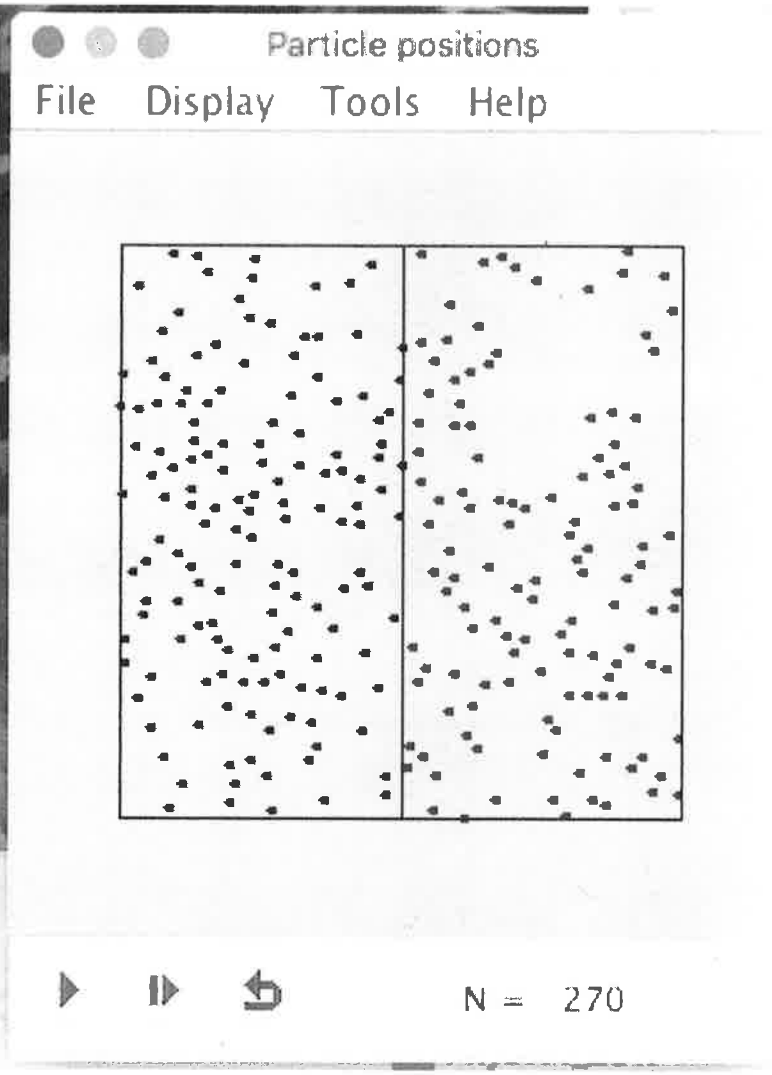
Particle positions
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N = 270

12. G & T 1.10'

Problem 1.10. Show that if the number of particles on the left-hand side of the box at $t = 0$ is equal to $n(0)$ rather than $N/2$, the solution of (1.7) is

$$\overline{n}(t) = \frac{N}{2} - \frac{N}{2}\Big[1 - \frac{2n(0)}{N}\Big]e^{-2t/N}. \qquad (1.9)$$

Note that $\overline{n}(t) \to N/2$ as $t \to \infty$ independent of the value of $n(0)$.

Eq. (1.7) is $\frac{\Delta \bar{n}}{\Delta t} \to \frac{d\bar{n}}{dt} = 1 - \frac{2\bar{n}(t)}{N}$

which with IC's $\bar{n}(0) = N$ has sol'n

$$\bar{n}(t) = \frac{N}{2}\left[1 + e^{-2t/N}\right]$$

But for arbitrary $\bar{n}(t=0)$ which I'd like to call n_0, soln is found via

$$d\bar{n} = \left(1 - \frac{2}{N}\bar{n}\right)dt$$

$$\Rightarrow \frac{d\bar{n}}{\left(1 - \frac{2}{N}\bar{n}\right)} = dt \Rightarrow$$

$$\Rightarrow \left(-\frac{N}{2}\right)\frac{d\left(-\frac{2}{N}\bar{n}\right)}{\left(1 - \frac{2}{N}\bar{n}\right)} = dt$$

$$\Rightarrow -\frac{N}{2}\ln\left(1 - \frac{2}{N}\bar{n}\right) = t + C$$

$$\Rightarrow 1 - \frac{2}{N}\bar{n} = \tilde{C}e^{-2t/N}$$

$$\Rightarrow \bar{n} = \frac{N}{2} - \frac{N}{2}\tilde{C}e^{-2t/N}$$

Thus $\bar{n}(0) \equiv n_0 = \frac{N}{2}(1 - \tilde{C})$

$$\Rightarrow 1 - \frac{2}{N} n_0 = \tilde{C}$$

or

$$\bar{n}(t) = \frac{N}{2} - \frac{N}{2}\left(1 - \frac{2n_0}{N}\right) e^{-2t/N}$$ ☺

13: $PV = nRT \; = \; \underline{NkT}$

HI region has mass of $100 M_\odot$

$T = 50\,K$ (cold!)

$n = 500\ cm^{-3}$

To find: P and V

The number density is $\dfrac{N}{V} = 500\ cm^{-3}$

Thus $P = \dfrac{N}{V} kT$

$\frac{N}{V} = 5\times10^{9}/m^3$

$= \left(500\ \frac{1}{cm^3} \times \frac{1\,cm^3}{10^{-6}\,m^3}\right) kT$

$k = 1.38\times10^{-23}\ J/K$, $T = 50\,K$

$\Rightarrow\ P = 3.45\times10^{-13}\ \underbrace{N/m^2}_{\equiv\, Pa}$

Now $V = \dfrac{V}{N} N$

where $N = \dfrac{100 M_\odot \times \dfrac{1.99\times10^{30}\ kg}{1\ M_\odot}}{1.67\times10^{-27}\ kg}$

$N = 1.19\times10^{59}$ atoms

$\Rightarrow\ V = \dfrac{1}{5\times10^{8}/m^3} \cdot 1.19\times10^{59} = \underline{2.38\times10^{50}\ m^3}$

Since $\exists$ $9.461\times10^{15}\ m/ly$

$V = \dfrac{2.38\times10^{50}\ m^3}{\left(9.461\times10^{15}\ \frac{m}{ly}\right)^3}$

$\underline{V = 281.5\ ly^3}$