Swarthmore College
Statistical Physics Honors Exam
Spring 2014

Name: ____

This is a 3-hour closed-book / closed-note exam to be printed on 8½ x 14 in^2 legal paper. You may use pen/pencil, ruler, non-programmable calculator, and the information sheet attached at the end. Each sub-question is worth 5 points. To earn full credit, you must show your work. Partial credit will be awarded for correctness and clarity of physical understanding. Pictures and sentences help. Cross out or erase incorrect first attempts, or else you may be penalized even if one of your attempts is correct. Use both sides of each page as needed. Good luck!

PROBLEM	SCORE
1.	/10
2.	/15
3.	/15
4.	/15
5.	/15
6.	/15
7.	/10
8.	/5
Total	/100

[1] Consider a "baby Einstein" solid of N "oscillators", where each "oscillator" has only two possible energy levels, 0 and ε.
(a) Sketch your expectation for the total energy and the heat capacity versus $k_B T$. Label salient features.
(b) Compute the total energy U versus temperature for the case $N \gg U/\varepsilon \gg 1$.

[2] An ideal diatomic gas at $\{p_0, V_0, T_0\}$ is compressed isobarically to half its initial volume. The number *n* of moles is held constant. T_0 is near room temperature.

(a) Explain how this could be done in the lab, and why it is irreversible.

(b) On a p-V diagram, sketch a two-step reversible path between the same initial and final states; label each step by name and "fast" or "slow".

(c) Find the minimum volume on the reversible path you just constructed.

[3] Consider a spin-1 paramagnet of N spins in a magnetic field B, at temperature T. There are three allowed energy levels for each spin: $-\mu B$, 0, $+\mu B$.

(a) Find the average number of spins aligned with the field.

(b) Find the average energy of a spin.

(c) Find the number of microstates with $\{n_+, n_0, n_-\}$ spins in the respective energy levels.

[4] Consider a system of identical spin-up electrons in a trap where the allowed energy levels are evenly spaced starting at zero: $E_n = n\varepsilon$ where $n=\{0, 1, 2, 3,\ldots\}$ and ε is the level spacing. For the following questions, let **N=5** be the number of electrons and let q be the number of energy units the system has in excess of the ground state energy.

(a) Construct the ground state, q=0.

(b) Enumerate all possible system states for q=3.

(c) Estimate the chemical potential and the temperature, for q=3.

[5] The multiplicity of an Einstein solid of N oscillators with q>>N units of energy is $\Omega=(eq/N)^N$.
(a) Find the multiplicity of two Einstein solids, each of N oscillators, sharing q energy units.
(b) Sketch your answer to (a) versus the energy in one of the solids, and indicate salient features.
(c) Use this sketch to discuss the second law of thermodynamics.

[6] Consider all the ways for 3 identical particles to be distributed amongst N different zero-energy states. In particular, find the partition function of the system…

(a) …from the classical single-particle partition function.

(b) …if the particles are fermions.

(c) …if the particles are bosons.

[7] A heat engine with efficiency of 1/3 does useful work at rate of 2 kW.
(a) How much heat is rejected in one minute?
(b) If the cold reservoir is at 20 C, what is the range of possible temperatures for the hot reservoir?

[8] Find the total energy of the thermal photons present in a two-dimensional square box of area L^2 at temperature T.

(scrap page)

I. MATHEMATICS

$\binom{N}{n} = \frac{N!}{n!(N-n)!}$
for $n \gg 1$:
$n! \approx n^n e^{-n}\sqrt{2\pi n}$
$\ln(n!) \approx n\ln(n) - n$
$\mathrm{d}\ln(n!)/\mathrm{d}n \approx \ln(n)$
$\exp(x) = 1 + x + x^2/2 + \ldots$
$\ln(1+x) = x - x^2/2 + \ldots$
$(1+x)^n = 1 + nx + \frac{1}{2}n(n-1)x^2 + \ldots$
$\sin\theta = \theta - \theta^3/6 + \ldots$
$\cos\theta = 1 - \theta^2/2 + \ldots$
$\tan\theta = \theta + \theta^3/3 + \ldots$
$\sinh\theta = \theta + \theta^3/6 + \ldots$
$\cosh\theta = 1 + \theta^2/2 + \ldots$
$\tanh\theta = \theta - \theta^3/3 + \ldots$
$\int_0^\infty x^n e^{-x}\mathrm{d}x = n!$

n	$\int_0^\infty x^n e^{-x^2}\mathrm{d}x$	$\int_0^\infty [x^n/(e^x-1)]\mathrm{d}x$
0	$\sqrt{\pi}/2$	∞
1/2	0.6127	2.315
1	1/2	$\pi^2/6$
3/2	0.4532	1.783
2	$\sqrt{\pi}/4$	2.404
3	1/2	$\pi^4/15$
4	$3\sqrt{\pi}/8$	24.89

II. PHYSICAL CONSTANTS

$k_B = 1.381 \times 10^{-23}$ J/K
$R = 8.315$ J/(mol $\cdot$ K)
$N_A = 6.022 \times 10^{23}$ mol^{-1}
$e = 1.602 \times 10^{-19}$ C
$k_e = 1/(4\pi\epsilon_o) = 9.0 \times 10^9$ N $\cdot$ m^2/C^2
$\epsilon_o = 8.854 \times 10^{-12}$ C^2/(N $\cdot$ m^2)
$\mu_o = 4\pi \times 10^{-7}$ T $\cdot$ m/A
$c = 1/\sqrt{\epsilon_o\mu_o} = 2.998 \times 10^8$ m/s
$h = 6.626 \times 10^{-34}$ J $\cdot$ s
$\hbar = h/(2\pi) = 1.055 \times 10^{-34}$ J $\cdot$ s
$m_e = 9.1094 \times 10^{-31}$ kg $= 0.511$ MeV/c$^2 = 0.000549u$
$m_p = 1.6726 \times 10^{-27}$ kg $= 938.3$ MeV/c$^2 = 1.007280u$
$m_n = 1.6605 \times 10^{-27}$ kg $= 939.6$ MeV/c$^2 = 1.008665u$
$\alpha = k_e e^2/(\hbar c) = 1/137$
$r_o = \hbar/(\alpha m_e c) = 5.29 \times 10^{-11}$ m $= 0.529$ Å
$\mu_B = e\hbar/(2m_e) = 9.274 \times 10^{-24}$ J/T
$\sigma = 5.670 \times 10^{-8}$ W/(m$^2 \cdot$ K^4)
$\lambda_P T = 2900$ μm $\cdot$ K
solar constant = 1370 W/m^2
$G = 6.673 \times 10^{-11}$ N $\cdot$ m^2/kg^2
$g = 9.8$ m/s^2

III. CONVERSIONS

1 eV $= 1.602 \times 10^{-19}$ J
1 cal = 4.186 J
1 BTU = 1055 J
1 kWh $= 3.6 \times 10^6$ J
1 hp = 746 W = 550 ft $\cdot$ lb/s
1 u $= 1.6605 \times 10^{-27}$ kg = 931.5 MeV/c^2
1 atm $= 1.013 \times 10^5$ Pa = 14.7 lb/in^2 = 760 mmH
1 cP = 0.01 g/(cm·s) = 0.001 Pa·s = 1 mPa·s
$T_K = T_C + 273.15$ K
$T_F = (9/5)T_C + 32$ F
$nR = Nk_B$
$\gamma = 1 + 2/f$