Physics 112: Classical Electromagnetism, Fall 2013
Seminar 13

Material summary:

• In the last seminar Peter has shown how electromagnetism can be 'naturally' described in four-dimensional space time. In this seminar we will see how electromagnetism provided Einstein the scaffolding upon which he built special relativity. As promised at the beginning of the semester, we will go from $1/r^2$ to $E = mc^2$.

• For this seminar you are asked to read Einstein's original 1905 paper on special relativity. You are also asked to read a followup paper he wrote in which he showed that special relativity implies the equivalence between mass and energy through one of the most celebrated and feared equations of the 20th century: $E = mc^2$.

• In this seminar we will also explore some of the finer theoretical aspects of electromagnetism: in particular we will see that electromagnetism can be derived from an action principle (in the same way that the motion of a mechanical particle can be derived from an action). We will also see that the ability to change the 'gauge' (known as 'gauge invariance') naturally leads to charge conservation. This meshes with something that you have discussed in 111: *whenever a quantity is conserved it is associated with a symmetry.*

Reading:

• Einstein's 1905 paper proposing his theory of special relativity, *On the Electrodynamics of Moving Bodies.*

> You should skim all of the paper, but read in detail only a few parts which I will list here: read the introduction, Part I (Kinematics) Sec. 1 and Sec. 2; Part II (Electrodynamical part) Secs. 6, 7 and 8.

• Einstein's 1905 paper deriving the equation $E = mc^2$.

• Notes on the calculus of variations.

Snacks!:

Presentations:

• **Mike and Laura:** *What is a scattering cross-section?*

From our discussion last seminar it was clear that Mike and Laura presented different ways of looking at the scattering cross-section. I want us to discuss this question a bit more. I will set up the problem again then I would like for Mike and Laura to re-state their ways of explaining the physical meaning of a scattering cross-section and for us to discuss how these perspectives differ.

- **Jamie:** *Approximations made in the electromagnetic radiation from a dipole*

Present the three approximations made when discussing the electromagnetic radiation from a dipole.

- **Stefan:** $1/r^2$

Talk to us about in what sense the $1/r^2$ law underlies all of electromagnetism. Where have we seen $1/r^2$ behavior? Under what circumstances have been explored in class in which $1/r^2$ is modified?

- **Adrian:** $E = mc^2$.

Present Einstein's derivation of the equation $E = mc^2$.

Problems:

Give short answers to the following questions which pertain to Einstein's original 1905 paper on special relativity:

• What is Einstein's justification of his statement in the introduction that "light is always propagated in empty space with a definite velocity c which is indecent of the state of motion of the emitting (or receiving) body."?

• For Einstein, what is a "rigid body"?

• Draw a space-time diagram which describes length contraction as discussed in section 2.

• On a space-time digram prove Einstein's statement "So we can see that we cannot attach any *absolute* signification to the concept of simultaneity....".

• At the beginning of Part II, section 6 Einstein writes down Maxwell's field equations in vacuum. He uses non-standard notation; translate his notation to the notation we've been using during this semester. Also, he does not write down *all* of the equations– which (two) equations is he missing?

• Show that the first Maxwell equation

$$\frac{1}{c}\frac{\partial X}{\partial t} = \frac{\partial N}{\partial y} - \frac{\partial M}{\partial z}, \tag{1}$$

becomes

$$\frac{1}{c}\frac{\partial X}{\partial \tau} = \frac{\partial}{\partial \eta}\left\{\beta\left(N - \frac{v}{c}Y\right)\right\} - \frac{\partial}{\partial \zeta}\left\{\beta\left(M + \frac{v}{c}Z\right)\right\} \tag{2}$$

where τ is the time coordinate according to an observer who is moving relative to the coordinate system at rest with speed v along their common x-axis and

$$\beta \equiv \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{3}$$

Make sure you understand how the multivariate chain rule works and how it applies to this situation (see this link for further details)!

• After he writes the field equations written in terms of τ, ξ, η, ζ describe (in words) how Einstein uses the principle of relativity in order to infer how the components of the electric and magnetic fields transform between different inertial frames.

• Describe how Einstein's 'law of aberration' (written down in Sec. 7) explains stellar aberration and therefore removes any need to invent a 'luminiferous ether'.

• From Sec. 8 comment on the importance of Einstein's observation that "It is remarkable that the energy and frequency of a light complex vary with the state of motion of the observer in accordance with the same law."

Speed of light through flowing water– The Fresnel/Fizeau experiment.

Imagine you are in the lab frame and there is a tube of water which is flowing at a speed v relative to you. What is the speed of light (traveling the same direction as the flow) inside of the water under the assumption that $v \ll c$. Compare this to the ether-drag proposed by Fresnel in order to explain stellar aberration observed through a telescope filled with water (see Eq. 12 in the notes on ether from Seminar 11).

Use the Lorentz transformation between the lab frame and the rest frame of the flowing water

$$
\begin{aligned}
t' &= \gamma\left(t - \frac{v}{c^2}x\right), && (4)\\
x' &= \gamma\,(x - vt)\,, && (5)\\
y' &= y, && (6)\\
z' &= z. && (7)
\end{aligned}
$$

in order to transform the speed of the light in the rest frame of the water, $u' = dx'/dt' = c/n$, to the speed relative to the lab frame, $u = dx/dt$.