# Physics 112: Classical Electromagnetism, Fall 2013
# Midterm 1

**Note**: I have written this exam so that it should take no longer than two and half hours to complete. However, you may take as much time as you need to write up solutions to these questions. Take the exam in one sitting with (short) breaks. Please note the amount of time it took for you to complete the exam.

This is a closed book exam. You are allowed to use Mathematica (or equivalent) for symbolic manipulations, but no problem requires its use. Please note if you used Mathematica (or equivalent) when deriving a result.

Show all work, and take particular care to explain what you are doing. Please use the symbols described in the problems, define any new symbols that you introduce, and label any drawings that you make.

**Make sure to show all of your work- it is your job to demonstrate your knowledge in the answers to these questions!**

1. *Parallel and slanted plate capacitors.* In this problem you will consider the capacitance of two conducting plates.

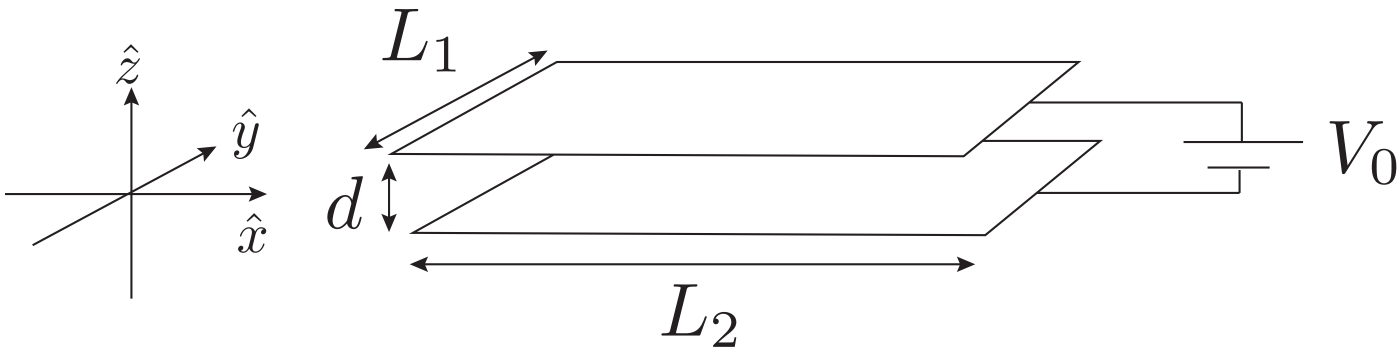


Figure 1: Two parallel conducting plates held at a potential difference $V_0$.

**A.** First consider a parallel plate capacitor, as shown in Fig. 1. Ignoring edge effects, use the Laplace equation in Cartesian coordinates to determine the potential between the plates and from this the electric field between the plates. (*Hint:* due to the approximate symmetry of the problem, the potential does not depend on $x$ or $y$.)

**B.** Use the energy stored in the electric field to derive an expression for the capacitance of the system. *Hint*: express the energy in terms of the electric field as well as in terms of the capacitance, $C$, and potential difference $V_0$.

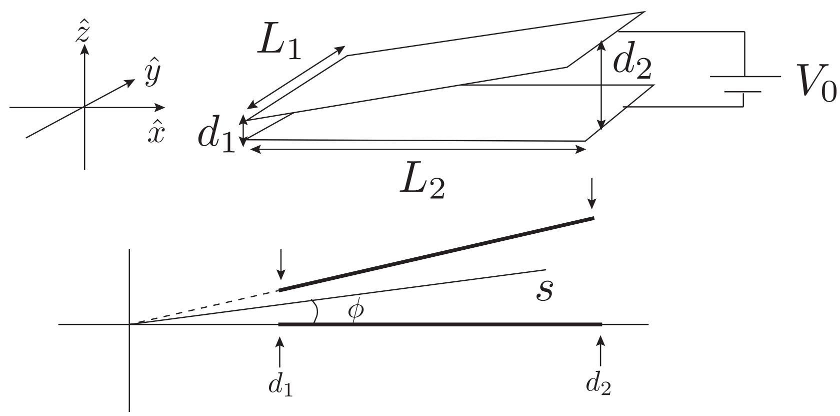


Figure 2: Two slanted conducting plates held at a potential difference $V_0$.

**C.** Now consider two conducing plates which are slanted and held at a potential difference $V_0$. Again, ignoring edge-effects, use the Laplace equation in *cylindrical* coordinates to determine the potential between the plates, and from this the electric field between the plates. (*Hint:* as with part (A), due to the approximate symmetry of the problem, note that the potential does not depend on $s$ or $z$.)

You will need to know the gradient in cylindrical coordinates is given by

$$\vec{\nabla} t = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}, \tag{1}$$

the Laplacian is given by

$$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}, \tag{2}$$

and the volume element is

$$d\tau = s ds d\phi dz. \tag{3}$$

**D.** As in part (B), use the energy stored in the electric field to derive an expression for the capacitance of the system. *Hint*: as before, express the energy in terms of the electric field as well as in terms of the capacitance and potential difference between the plates.

**E.** Give an explanation as to why the capacitance is increased by $\epsilon_r$ when the space between the conductors is filled with a linear dielectric with dielectric constant $\epsilon_r$. Explicitly show that this is true for a parallel plate capacitor.

2. *Method of images.* In this problem you will consider the induced charge distribution due to a point charge, $q$, outside of a solid conducting sphere.

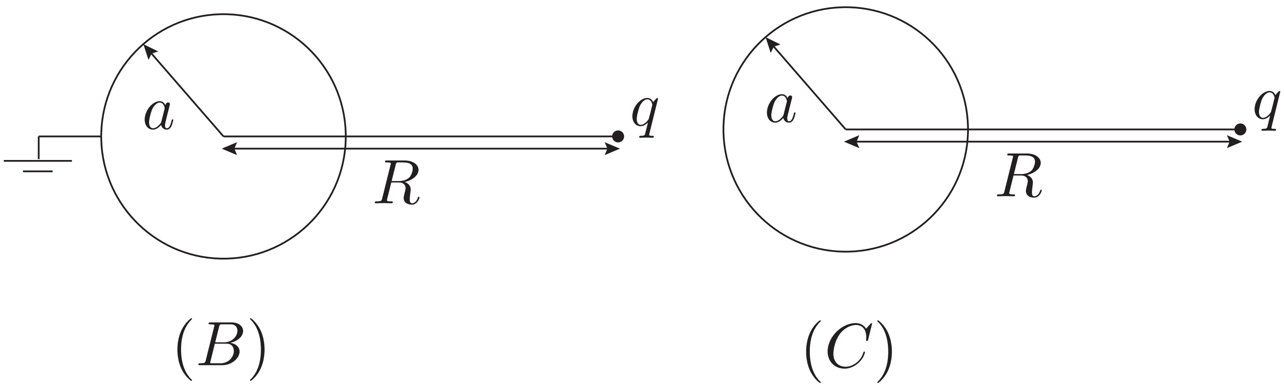


Figure 3: The two conducting spheres which are analyzed in parts (A) and (C) of this problem.

**A.** Briefly justify why we are able to use the method of images when solving certain electrostatic problems– i.e., what must be true in order to use this method?

**B.** Suppose that the sphere is grounded and in the presence of a point charge outside of the sphere. Use the method of images to find the electrostatic potential on the surface of the sphere. From this determine the force on the charge in terms of $q$, $R$, and $a$. Be sure to clearly state the boundary conditions on the electrostatic potential.

**C.** Next, consider an insulated conducting sphere with no net charge. Determine the expression for the force acting on the external point charge. (*Hint:* remember that the total charge on the conducting sphere must add to zero. Using the image charge configuration from part (A) where can you add additional charge but keep the potential within the conducting sphere constant?)

Perform one check that your answer makes sense.

**D.** Write down an expression for the surface charge density for part **C**. Perform one check to make sure that your answer makes sense. You are free to choose any check you want, including making a plot.

To answer this question you will need the relation between Carestian and spherical coordinates:

$$\begin{aligned} x &= r\cos(\phi)\sin(\theta), & (4)\\ y &= r\sin(\phi)\sin(\theta), & (5)\\ z &= r\cos(\theta). & (6) \end{aligned}$$

*Note*: first use the equations of electrostatics to show that for the potential produced by the image charges,

$$\frac{\partial V_{\text{image}}}{\partial n} = -\frac{\sigma}{\epsilon_0}, \tag{7}$$

where $n$ is the direction normal to the conductor's surface.

3. *Short questions*. These questions require very little direct calculation. I am looking for a clear physical reasoning.

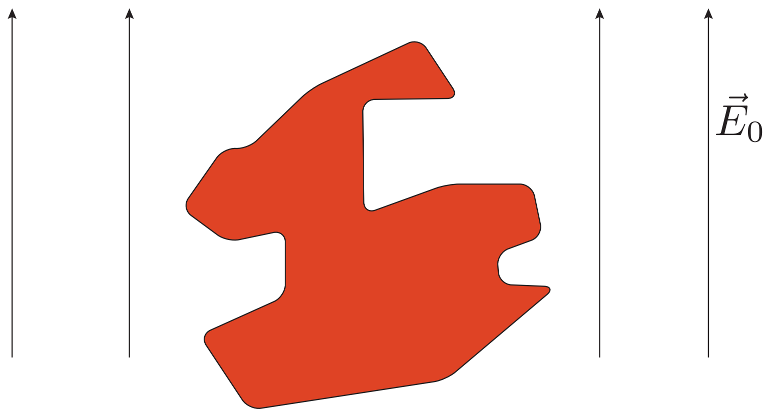


Figure 4: Diagram for question 3A.

**A.** If a irregularly shaped piece of solid dielectric material is placed in a uniform external electric field, does it feel a net force? Explain your reasoning.

**B.** Imagine I asked you the question: "A linear dielectric sphere is placed within an external field $\vec{E}_0$ with no free charge. What is the electric field inside of the sphere?" Explain whether you can use separation of variables in this problem. If you can't, what piece of information could I supply which would allow its use?

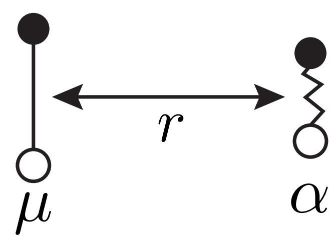


Figure 5: Diagram for question 3C.

**C.** Imagine we bring an object with an intrinsic dipole moment $\mu$ towards an object with a polarizability $\alpha$. How does the force between them scale with their separation, $r$? How does the energy of their interaction scale with their separation? This is called a 'dipole/induced-dipole' interaction.