Mankiw (1986) on the equity premium and the concentration of shocks
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1. Introduction

It turns out that there is absolutely no way to 'reverse engineer' the connection of Romer's problem 8.11 to Mankiw's argument about the equity premium, without reading Mankiw's paper. The problem is that Mankiw (1986) was really not writing about the equity premium *per se*. He was interested, instead, in showing that the 'representative consumer' model that was used, among other things, to estimate the degree of risk aversion using data on aggregate consumption and financial returns, was invalid under some realistic circumstances. The circumstances he emphasized were a combination of *incomplete financial markets* and a *positive third derivative* of the utility function. Applying his argument to the equity-premium puzzle, he developed an example in which the representative-consumer calculation would generate far too high an estimated degree of risk aversion. In his example, a macroeconomist applying the representative-consumer model to observed data on aggregate consumption and the equity premium would infer a very high degree of risk aversion even though the underlying level of risk-aversion was modest.

Mankiw's argument reveals a lot about using and interpreting the consumption-based asset-pricing model. He notes first that if financial markets are complete, any two consumers who are identical in period 0 will acquire a portfolio that will give each of them exactly the same state-contingent future consumption pattern. Under these conditions, it is harmless to aggregate consumers into a representative household for analytical purposes. If financial markets are incomplete, however, then even consumers who are identical *ex ante* may end up with different state-contingent consumption patterns *ex post*. In this case aggregation may produce misleading results.

To develop this point, Mankiw makes specific assumptions about uncertainty in aggregate consumption and about the differences in *ex post* individual consumption across a set of *ex ante* identical households. He takes these differences as given in equilibrium. He then derives the equilibrium returns on two assets that he assumes are freely tradable *ex ante*: risk-free debt and risky equity. He uses these returns to calculate the implied equity premium. He then returns to the representative consumer world to ask what this equilibrium equity premium, in combination with the aggregate consumption pattern, implies about the degree of risk aversion. He shows that the representative consumer apparatus works fine if utility is quadratic (so that certainty-equivalence holds), but that it produces a biased estimate of the degree of risk aversion if the utility function has a positive third derivative. The direction of the bias can in general be either way, but when the aggregate shock is an adverse one and the equity claim has a payoff that is positively correlated with aggregate consumption, the bias is positive: the representative-consumer approach <u>over</u>-estimates the degree of risk aversion, and by an increasing amount the more concentrated is the aggregate shock. In this world, the economist might infer a coefficient of relative risk aversion on the order of (say) 25 when the true coefficient (remember, all households have the same preferences) is between 2 and 5.

It is impossible to get any sense of this argument from Romer's problem 8.11, so here are is an outline of the full argument.

2. Defining the equity premium.

Suppose that from the perspective of period 0, there are two possible states of the world in period 1: a good state with probability ½ and a bad state with probability ½. Households can purchase or sell equity claims in period 0. These equity claims pay off $z > 1$ in the good state and zero in the bad state. Households can also buy or sell bonds, which are promises to pay an amount 1 in both future states.

Let's define the equity premium as π, where

$$1 + \pi = \frac{E[1+r_E]}{1+r_B}. \tag{1}$$

We can derive an expression for π using what we know about expected returns. The bond is riskless, so its gross real return satisfies $1 + r_B = 1/P_B$. The equity claim is risky; its expected gross real return satisfies $E[1 + r_E] = z/P_E$. The equity premium therefore satisfies

$$1 + \pi = \frac{p \cdot z}{2}. \tag{2}$$

where $p = P_B/P_E$ is the relative price of the bond in terms of the equity claim. In other words, given that the state-contingent payoffs in period 1 are known in period 0, there is an inverse relationship between the equity premium and the relative price of the equity claim.

3. Using the equity premium to estimate the degree of risk aversion

Our consumption Euler equation holds for any asset that can be freely purchased or sold. This implies that $E_0\left[\left(R_{i1} - R_{j1}\right) \cdot u'(C_1)\right] = 0$ for the gross real returns on any two assets i and j (make sure you are comfortable with this). Similar logic implies that the covariance between C_1 and the gross real return on any portfolio that can be acquired at a cost of zero in period 0 must be zero: $E_0[R \cdot u'(C_1)] = 0$. If households have identical consumption in period 1, we can use the latter condition to estimate the degree of risk aversion, given information about the return pattern and the pattern of aggregate consumption.

The case of the representative household

Consider issuing a bond in period 0 and using the proceeds to buy equity. This portfolio costs zero. Its payoffs are -1 in the bad state and $pz - 1 = 1 + 2\pi$ in the good state. Suppose further that households are identical *ex post*; per-capita consumption is 1 in the good state and $1 - \phi$ in the bad state, where $0 < 1 - \phi < 1$. The covariance condition can then be written

$$\left(\frac{1}{2}\right)(1 + 2\pi)u'(1) + \left(\frac{1}{2}\right)(-1)u'(1 - \phi) = 0. \tag{3}$$

This yields an equilibrium equity premium of

$$\pi = \frac{u'(1-\phi) - u'(1)}{2u'(1)}. \tag{4}$$

To estimate the degree of risk aversion in one simple case, suppose that utility is CRRA, so that $u(C) = C^{1-\theta}/(1 - \theta)$. In this case, equation (4) implies

$$\theta = -\log\left(\frac{1+2\pi}{1-\phi}\right) \cong \frac{2\pi}{\phi}. \tag{5}$$

Notice that equation (5) is a relationship between 3 variables, any two of which suffice to determine the third. In terms of the market equilibrium, ϕ and θ can be thought of as determining the equity premium. From the perspective of the macroeconomist, ϕ and π are observable data that together allow us to infer θ.

The case of ex post *heterogeneous households*

Suppose now that households are heterogeneous *ex post*. In particular, the entire reduction in aggregate consumption falls on a randomly-chosen fraction λ of households, where $\phi < \lambda \leq 1$. The covariance condition now takes the form

$$\left(\frac{1}{2}\right)(1+2\pi)u'(1)+\left(\frac{1}{2}\right)\lambda(-1)u'\big(1-(\phi/\lambda)\big)+\left(\frac{1}{2}\right)(1-\lambda)(-1)u'(1)=0,$$

so that the equilibrium equity premium is

$$\pi=\frac{\lambda[u'(1-(\phi/\lambda))-u'(1)]}{2u'(1)}. \tag{6}$$

Note that (6) reduces to (4) when $\lambda = 1$. Furthermore, Mankiw shows that $d\pi/d\lambda = 0$ if utility is quadratic, implying that in the certainty-equivalence case, relative asset prices are independent of the concentration of the aggregate shock. It follows that in this case, equation (5) will be reliable even when the conditions of the representative-consumer model do not hold *ex post*. The macroeconomist can use (5) to 'forecast' the equity premium given the degree of risk aversion, or to infer the degree of risk aversion from the observed equity premium, and in either case the results will be accurate.

If the utility function has a positive third derivative, however, equation (6) implies $d\pi/d\lambda < 0$: an increase in the concentration of the bad outcome (a reduction in λ) increases the equity premium. Equivalently, given equation (2), an increase in the concentration of the adverse shock makes equity even less attractive than it already is (being positively correlated with aggregate consumption), and reduces its relative price. In fact, Mankiw shows that

$$\lim_{\lambda\to\phi}\pi=\infty,$$

which says that the premium gets arbitrarily large as a given macro shock is concentrated on a smaller and smaller group of households. Crucially, this effect takes place for a given utility function and therefore a given degree of risk aversion. In this world, the relationship in equation (5) no longer yields an accurate estimate of the degree of risk aversion. *In particular, Mankiw shows that if the negative shock is highly concentrated, the degree of risk aversion that is implied by the representative-consumer model, using (5), may be an order of magnitude higher than the true degree of risk aversion.*

References

Mankiw, N. Gregory (1986) "The Equity Premium and the Concentration of Aggregate Shocks" *Journal of Financial Economics* 17: 211-219