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The Development of Arithmetic and Word-Problem Skill Among Students with Mathematics Disability

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Mathematics disability (MD) creates life-long challenges (Rivera-Batiz, 1992) for 5%–9% of the population (e.g., Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996). This makes prevention, which has been shown to substantially improve mathematics outcomes (e.g., Fuchs, Fuchs, Yazdian, & Powell, 2002; Griffin, Case, & Siegler, 1994), critical. Nevertheless, no intervention is effective for all students. Fuchs et al. (2005), for example, showed that a first-grade prevention program was highly efficacious at reducing the prevalence of MD at the end of first grade, with effects maintaining one year after tutoring ended (Compton, Fuchs, & Fuchs, 2009). Yet, 3%–6% of the school population continued to manifest severe mathematics deficits. Because we cannot expect prevention activities to be universally effective, the need for intensive remedial intervention persists even when strong prevention services are in place.

In this chapter, we focus on the remediation of mathematics deficits. Our emphasis is on third grade when serious mathematics deficits are clearly established and identification of MD often begins (Fletcher, Lyon, Fuchs, & Barnes, 2007). We focus on arithmetic and word problems because they represent two major dimensions of the mathematics curriculum in the primary grades. We begin by providing background on these two aspects of mathematical cognition. We then summarize the literature on the remediation of arithmetic and word-problem deficits. Next, using this literature, we derive principles for effective remediation and illustrate these principles with one remedial tutoring protocol. Finally, we discuss salient issues concerning MD and its remediation.

Development of and Distinctions between Arithmetic and Word-Problem Skill

Arithmetic refers to simple computation problems (e.g., 5 + 6 = 11; 12 – 5 = 7) that cannot be solved via algorithms. To answer arithmetic problems, mathematically competent individuals, including children and adults, use a mix of counting strategies, decomposition strategies, and automatic retrieval of answers from long-term memory. Consensus exists that arithmetic is essential (Kilpatrick, Swafford, & Findell, 2001), and research shows that arithmetic fluency is a significant path to procedural calculation and word-problem skill (Fuchs, Fuchs, Compton, et al., 2006). In developing arithmetic fluency, typical children develop procedural efficiency with counting. First they count two sets (e.g., 4 + 5) in their entirety (i.e., 1, 2, 3, 4, 5, 6, 7, 8, 9); then they count from the first addend (i.e., 4, 5, 6, 7, 8, 9); and eventually they count from the larger addend (i.e., 5, 6, 7, 8, 9). As conceptual knowledge about number becomes more sophisticated, they also develop decomposition strategies for deriving answers (e.g., [4 + 4 = 8] + 1 = 9). As increasingly efficient counting and decomposition strategies facilitate consistent and quick pairing of problems with correct answers, associations become established in long-term memory, and students gradually favor memory-based retrieval of answers (Ashcraft & Stazyk, 1981; Geary, Widaman, Little, & Cormier, 1987; Goldman, Pellegrino, & Mertz, 1988; Groen & Parkman, 1972; Siegler, 1987).

Students with MD manifest greater difficulty with counting (Geary, Bow-Thomas, & Yao, 1992; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007); they persist with immature back-up strategies (Geary et al., 2007); and they fail to make the shift to memory-based retrieval of answers (Fleishner, Garnett, & Shepherd, 1982; Geary et al., 1987; Goldman et al., 1988). When children with MD do retrieve answers from memory, they commit more errors and their retrieval speeds are less systematic than younger, typically developing children (Geary, Brown, & Samaranayake, 1991; Gross-Tsur et al., 1996; Ostad, 1997). Some researchers (e.g., Fleishner et al., 1982.; Geary et al., 1987; Goldman et al., 1988) consider arithmetic to be a signature deficit of students with MD, and difficulty with automatic retrieval of arithmetic facts is one of the most consistent findings in the MD literature (e.g., Cirino, Ewing-Cobbs, Barnes, Fuchs, & Fletcher, 2007; Geary et al., 2007; Jordan, Hanich, & Kaplan, 2003).

Arithmetic is incorporated into the curriculum at kindergarten through second grade, although many general educators do not explicitly promote arithmetic fluency (Miller & Hudson, 2007). Even so, typically developing students have considerable arithmetic fluency by third grade (Cirino et al., 2007), and when students still manifest deficiencies in third grade, a pressing need for remediation exists.

Less is known about typical development or about how students with MD come to develop difficulty with word problems than is known about competence with arithmetic. In contrast to arithmetic, where problems are already set up for solution, a word problem requires students to use text to identify missing information, construct the number sentence that incorporates the missing information, derive the calculation problem for finding the missing information, and finally solve that calculation problem. The need to use text to construct the problem model appears to alter the task, and some research suggests that calculations and word problems may represent distinct aspects of mathematical cognition (e.g., Fuchs, Fuchs, Compton, et al., 2006; Fuchs, Fuchs, Stuebing, et al., 2008; Swanson & Beebe-Frankenberger, 2004). If so, then calculation and word-problem skill would need to be considered separately in remediation.

The Remediation Literature

Arithmetic

Three major approaches for remediating arithmetic deficits have been documented in the literature: providing drill and practice, developing conceptual understanding to foster decomposition strategies, and teaching strategic counting. The literature, however, has focused heavily on drill and practice. Okolo (1992) and Christensen and Gerber (1990) contrasted computerized drill and practice in a game versus a drill format. Okolo found no significant differences between groups. Christensen and Gerber, by contrast, found that students were disadvantaged by the game format, perhaps due to its distracting nature. Tournaki (2003) contrasted paper-pencil drill and practice with a strategic counting condition. Results showed an advantage for strategic counting, but the validity of the study was compromised because the strategic counting condition incorporated stronger instructional principles than did the drill and practice condition. All three prior studies failed to include a control group to assess whether drill and practice effected better outcomes than business-as-usual schooling. Also, because this work was restricted largely to drill and practice, it does not contrast alternative forms of intervention and therefore fails to inform the nature of remediation. In addition, because participants in these prior studies had school-identified learning disabilities, it is unclear whether effects apply to students who experience mathematics difficulty.

To address these limitations, we recently extended this literature in a series of four studies in which we relied on random assignment, incorporated different approaches to remediation, included a control condition, and screened participants to ensure MD. In the first study (Fuchs, Powell, et al., 2008), our approach to remediation was drill and practice although we took an unconventional approach. Instead of simply requiring students to answer arithmetic problems, as typically done with drill and practice, we tried to ensure that students would practice correct responses. Each computerized drill and practice trial occurred as follows: Students saw a complete arithmetic problem "flash" briefly (i.e., 1.3 sec) and then reproduced the complete arithmetic problem (i.e., question stem and answer) from short-term memory. The assumption was that with repeated pairings of a question stem and its correct answer, the student would commit the arithmetic problem to long-term memory. Typically developing students achieve such automatic retrieval through repeated pairings, which occur naturally as students' counting strategies become more efficient and their back-up strategies become more sophisticated. Given the deficiencies of students with MD with counting and decomposition strategies, we decided to test the efficacy of the "direct route" for reliable and efficient pairings just described. We randomly assigned participants to four conditions that all relied on computer-assisted instruction with tutor supervision: arithmetic remediation, procedural computation-estimation remediation, remediation that combined arithmetic with procedural computation-estimation instruction, and word identification remediation (i.e., control). On arithmetic outcomes, only students who received arithmetic remediation outperformed those in the competing conditions. Effect sizes were large (0.69–0.78). We concluded that a "direct route" for drill and practice, which promoted reliable and efficient pairings of question stems with correct responses, was efficacious. Even so, we questioned whether a stronger focus on developing conceptual understanding to foster decomposition strategies, the second major approach to remediating arithmetic problems, might enhance learning.

Consequently, Powell, Fuchs, Fuchs, Cirino, and Fletcher (2009) randomly assigned students to four conditions: drill and practice as in Study 1; drill and practice as in Study 1 plus explicit conceptual instruction focused largely on decomposition strategies; procedural computation-estimation remediation; and control (no tutoring). The initial conceptual lessons focused on addition and subtraction concepts, adding/subtracting 0 and 1, and the commutative property of addition. Then, a tutor-directed lesson occurred whenever a new arithmetic family was introduced (every 3 to 6 sessions). The tutor focused the student's attention on how number sentences within the family are related and used manipulatives to teach strategies for decomposition in relation to the 10 set and in relation to doubles arithmetic problems (e.g., 2 + 2 = 4). Students also practiced decomposition strategies with number line flash cards (students derived equations for depictions of arithmetic problems on a number line that delineated the 10 set) and generated arithmetic problems for a family in a fixed time. The condition with conceptual lessons was, by necessity, longer than the condition that relied entirely on drill and practice. Despite more instructional time, effect sizes comparing each arithmetic remediation to the control condition were similar: 0.50 and 0.53. The same was true when comparing each NC remediation to the procedural computation-estimation remediation: 0.31 and 0.37. This suggests that explicit conceptual instruction to help students develop decomposition strategies for solving arithmetic problems does not impart added value over a direct route for intensive drill and practice.

We next turned our attention to the third major approach for remediating arithmetic deficits: teaching strategic counting (Fuchs, Powell, Seethaler, et al., 2009). Although students are not explicitly taught strategic counting in school, typically developing students (but not students with MD) discover these strategies on their own (Ashcraft & Stazyk, 1981; Geary et al., 1987; Goldman et al., 1988; Groen & Parkman, 1972; Siegler, 1987). With inefficient counting strategies, MD students pair question stems with answers slowly, taxing short-term memory, and their answers are often incorrect. Long-term representations for automatic retrieval of arithmetic problems therefore fail to establish correctly. It is also possible that students with MD have special difficulty committing arithmetic problems to memory. We hoped that explicit instruction on counting strategies would build arithmetic fluency (even if students remained incapable of automatic retrieval). We contrasted two conditions that incorporated strategic counting. One combined strategic counting with intensive drill and practice (as in Studies 1 and 2). The other, which was embedded in word-problem remediation, taught the same strategic counting, but practice with arithmetic problems was confined to 4-6 min each session.

In this third study, we randomly assigned students to three conditions: strategic counting arithmetic remediation plus drill and practice as in Studies 1 and 2; word-problem remediation that incorporated strategic counting (without the drill and practice used in Studies 1 and 2); and control. Both remediations effected superior arithmetic fluency compared to the control group (effects sizes: 0.52 and 0.58). The comparability of outcomes for the two remediation groups is notable because the condition that incorporated drill and practice allocated dramatically more time to arithmetic over the 48-session intervention: 20–30 min per session versus 4–6 min per session. We, therefore, conclude that teaching students strategic counting, while providing frequent but brief practice to gain efficiency in using those counting strategies, results in arithmetic fluency that is comparable to an expanded arithmetic remediation that is devoted entirely to arithmetic and that also incorporates the drill and practice used in Studies 1 and 2. Study 3 results suggested promise for the strategic counting remediation.

In our fourth study (Fuchs, Powell, Seethaler, et al., 2010), we extended the Study 3 findings by assessing the effects of strategic counting instruction, with and without deliberate practice with those counting strategies, on arithmetic fluency. We contrasted a no-tutoring control group against two variants of strategic counting instruction. Both were embedded in word-problem remediation. In one variant, the focus on arithmetic was limited to a single lesson that simply taught the counting strategies (i.e., strategic counting instruction without deliberate practice). In the other variant, students were taught counting strategies in the same single lesson but then also practiced strategic counting for answering arithmetic problems for 4-6 min each session (i.e., strategic counting instruction with deliberate practice). Pinpointing the value of practice in this controlled way is important because although it is assumed necessary for students with MD, no studies had isolated its effects for this population of learners. Study 4 findings suggest its importance. The remediation condition that included deliberate practice with the counting strategies effected superior arithmetic fluency compared to the control condition, with a large effect size of 0.67. More important, students who received deliberate practice also outperformed those who were taught the counting strategies but were not provided with deliberate practice, with an effect size of 0.22. This effect size meets the federal What Works Clearinghouse criterion for effective practice.

Word Problems

The major approach in the research literature for developing word-problem skill for students with learning difficulties relies on schema theory, which is based on the concept of lateral transfer by which children recognize problems across numerous experiences to abstract generalized problem-solving strategies. Some refer to the abstraction of generalized problem-solving strategies as the development of schemas (Brown et al., 1992; Gick & Holyoake, 1983). A *schema* is a category that encompasses similar problems; it is a problem type (Chi, Feltovich, & Glaser, 1981; Gick & Holyoake, 1983; Quilici & Mayer, 1996). For example, word problems that describe parts being combined into a whole represent a "Total" problem type (e.g., John has 3 cats. He also has 5 dogs. How many pets does John have?); by contrast, the "Difference" problem type compares two quantities (e.g., John has 5 pets. His best friend has 2 pets. How many more pets does John have?). Instruction based on schema theory encourages students to develop a schema for each problem type. The broader the schema, the greater the probability students will recognize a novel problem as belonging to that familiar schema for which they know a solution method. With broader schemas, problem-solving performance improves. For example, a problem that belongs to a familiar problem type may appear novel (but still require a similar solution strategy) because it incorporates irrelevant information or relevant information outside of the problem narrative (e.g., in tables) or includes unusual vocabulary and so on. When students have broad schemas that systematically incorporate novel features, they know when to apply solution strategies, enhancing the word-problem performance. Broadening schemas should affect breadth of learning or transfer (Brown et al., 1992; Glaser, 1984).

To facilitate schema development, teachers must first teach problem-solution rules and then help students develop schemas for the problem types and awareness of those schemas (Cooper & Sweller, 1987). In the past decade, some research programs have relied on schema theory to design explicit instruction for enhancing word-problem skill. Jitendra and colleagues demonstrated acquisition, maintenance, and transfer effects for students with serious mathematics deficits or with risk for MD at eighth grade (Jitendra, DiPipi, & Perron-Jones, 2002), sixth grade (Xin, Jitendra, & Deatline-Buchman, 2005), and third and fourth grades (Jitendra et al., 2007; Jitendra et al., 1998; Jitendra & Hoff, 1996). In our intervention work, we have also relied on schema theory. Similar to Jitendra's schema-based strategy instruction, we teach students to understand the underlying mathematical structure of the problem type, to recognize the basic problem type, and to solve the problem type. In contrast to Jitendra, we incorporate a fourth instructional component, in keeping with Cooper and Sweller (1987), by explicitly teaching students to broaden those schema by learning about transfer features (e.g., irrelevant information; novel questions that require an extra step; relevant information presented in charts; combinations of problem types). In our work, we have addressed these and other transfer features. The addition of explicit instruction on transfer features should lead to more flexible and successful problem solving. We refer to the combination of all four instructional components as *schema-broadening instruction*, or SBI.

In our first randomized control study, Fuchs, Fuchs, Prentice, Burch, Hamlett, Owen, et al. (2003) isolated the effects of our fourth instructional component (explicitly teaching for transfer) from the first three instructional components (teaching students to understand the underlying mathematical structure of the problem type, to recognize the basic problem type, and to solve the problem type). Working with third graders without MD, we found that SBI (i.e., all four components) strengthened word-problem performance over and beyond experimenter-designed instruction on the first three instructional components. In a series of additional studies on SBI, also conducted in general education (Fuchs, Fuchs, Craddock, et al., 2008; Fuchs, Fuchs, Finelli, Courey, & Hamlett; 2004; Fuchs, Fuchs, Finelli, et al., 2006; Fuchs, Fuchs, Prentice, Burch, Hamlett, Owen, & Schroeter, 2003; Fuchs, Fuchs, Prentice, Hamlett, et al., 2004), effect sizes favoring SBI were large (0.89–2.14). Random assignment, however, occurred at the classroom level, with limited numbers of students with MD.

More recently, Fuchs, Seethaler, et al. (2008) piloted SBI, this time conducted as tutoring rather than whole-class instruction, for third graders whom we identified as having mathematics and reading difficulties (i.e., scoring on average at the 10th percentile in math and reading). The 35 participants were randomly assigned to receive SBI tutoring or to continue in their mathematics program without modification. Results favored the word-problem performance among the tutored students, but instructional time across the tutored and control students was not controlled, a limitation we addressed in our next study (Fuchs, Powell, Seethaler, et al., 2009), where we contrasted SBI tutoring not only to a control group but also to a contrasting tutoring condition. Results supported the efficacy of SBI tutoring in relation to the control group as well as the competing, active condition, and findings were replicated in Fuchs, Powell, Seethaler, et al., (2010).

General Principles for Effective Mathematics Remediation

In this section, we provide an overview of a set of principles for remediating arithmetic and word-problem deficits. Then, we illustrate the application of these principles using one remedial tutoring protocol.

Seven Research-Based Principles for Effective Remediation

The first principle of effective intervention for students with MD is *instructional explicitness*. Typically developing students profit from the general education mathematics program that relies, at least in part, on a constructivist, inductive approach to instruction. Students who accrue serious mathematics deficits, however, fail to profit from those programs in a way that produces understanding of the structure, meaning, and operational requirements of mathematics. A meta-analysis of 58 math studies (Kroesbergen & & Van Luit, 2003) revealed that students with MD benefit more from explicit instruction than from discovery-oriented methods. Therefore, effective intervention for students with MD requires explicit, didactic instruction in which the teacher directly shares the information the child needs to learn and systematically supports student mastery.

Explicitness is not, however, sufficient. A second and often overlooked principle of effective intensive mathematics intervention is *instructional design to minimize the learning challenge*. The goal is to anticipate and eliminate misunderstandings with precise explanations and with the use of carefully sequenced instruction so that the achievement gap can be closed as quickly as possible.

This is especially important given the ever-changing and multiple demands of the mathematics curriculum.

The third principle of effective intensive mathematics intervention is the requirement that instruction provide a *strong conceptual basis* for procedures. Special education is already strong in emphasizing *drill and practice*, a critical and fourth principle of effective practice. Yet, special education has sometimes neglected the conceptual foundation of mathematics, and such neglect can cause confusion, learning gaps, and a failure to maintain and integrate previously mastered content. In terms of drill and practice, we note that this practice needs to be rich in *cumulative review*, the fifth principle of effective intervention.

The sixth principle concerns the need to incorporate *motivators to help students regulate their attention and behavior and to work hard*. Students with learning disabilities often display attention, motivation, and self-regulation difficulties, which may adversely affect their behavior and learning (e.g., Fuchs et al., 2005, 2006). By the time students enter intensive intervention, they have experienced repeated failure, causing many to avoid the emotional stress associated with mathematics. They no longer try to learn for fear of failing. Therefore, intensive intervention must incorporate systematic self-regulation strategies and motivators; for many students, tangible reinforcers are required.

The seventh and final principle of remediation is the need for *systematic, ongoing progress monitoring* to gauge the effectiveness of a tutoring program for the individual student. No instructional method, even those validated using randomized control studies, works for all students. Because schools must assume that validated intervention protocols will work for most but not all students, schools need to monitor the effects of interventions on individual children's learning. That way, children who do not respond adequately can be identified promptly, and the teacher can adjust the intervention to develop an individually tailored instructional program. This leads us to a seventh essential principle of intensive remedial programming: ongoing progress monitoring. Teachers use progress monitoring to determine whether a validated treatment protocol is in fact effective for a given student. When progress monitoring reveals that a student is failing to respond as expected to a validated intervention protocol, progress monitoring is then used for a second purpose: to formulate an individually tailored instructional program that is in fact effective for that student.

Incorporating the Seven Research-Based Principles for Effective Remediation: A Sample Tutoring Protocol

To illustrate the use of the first six research-based principles for effective remediation, we describe a validated tutoring program called Pirate Math, designed to remediate arithmetic as well as word-problem deficits while building procedural calculation and algebra skill. We incorporate a pirate theme because within this schema-broadening instructional program, students are taught to represent the underlying structure of word problem types using algebraic equations. "They find X, just like Pirates find X on treasure maps." After we describe Pirate Math and explain how it incorporates the first six principles of effective remediation, we then explain how one special education teacher implemented Pirate Math in conjunction with systematic, ongoing progress monitoring, the seventh instructional principle, to assess the student's response to Pirate Math and to individualize the student's program as required.

How Pirate Math addresses the first six principles of effective remediation. Pirate Math comprises four units: an introductory unit, which addresses mathematics skills foundational to solving word problems, and three word-problem units, each focused on a different type of word problem. Pirate Math has been validated for use in small groups as a secondary prevention intervention; it has also been validated for one-to-one implementation at the tertiary prevention level. Every tutoring lesson is scripted, but scripts are studied; they are not read or memorized. Pirate Math runs for 16 weeks, with 48 sessions (3 per week). Each session lasts 20–30 min. The instruction, as outlined below, is *systematic and explicit*; it is designed with care to *minimize the learning challenge*; it is rich in ***concepts***; it incorporates *drill and practice* as well as *cumulative review*; and it relies on *systematic reinforcement* to encourage good attention, hard work, and accurate performance.

The introductory unit addresses mathematics skills foundational to word problems. Tutors teach a single lesson on strategic counting for deriving answers to arithmetic problems, review algorithms for double-digit addition and subtraction procedural calculations, teach methods to solve for "X" in any position in simple algebraic equations (i.e., $a + b = c$; $d - e = f$), and teach strategies for checking work within word problems.

The single strategic counting lesson is designed to remediate arithmetic deficits. Students are taught that if they "just know" the answer to an arithmetic problem, they "pull it out of their head." If, however, they do not know an answer immediately, they "count up." Strategic counting for addition and subtraction is introduced with the number line. For addition, the min strategy is taught: Students start with the bigger number and count up the smaller number on their fingers. The answer is the last number spoken. For subtraction, the missing addend counting strategy is taught, which requires new vocabulary. The *minus number* is the number directly after the minus sign. The *number you start with* is the first number in the equation. Students start with the minus number and count up to the number they start with. The answer is the number of fingers used to count up.

Practice in strategic counting is then incorporated in subsequent lessons. The tutor begins each session by asking the student, "What are the two ways to find an answer to a math fact?" The student responds, "Know it or count up." Then, the student explains how to count up an addition problem and how to count up a subtraction problem. Next, the tutor requires the student to count up two addition and two subtraction problems. Then, the tutor conducts a flash card warm-up activity, in which students have 1 min to answer arithmetic problems. If they respond incorrectly, the tutor requires them to count up until they derive the correct answer. At the end of 1 min. the tutor counts the cards, and the student then has another min to beat the first score. Also, throughout the lesson, whenever the student makes an arithmetic error, the tutor requires the student to count up. Finally, when checking the paper-pencil review, the tutor corrects arithmetic errors by demonstrating the counting strategy.

Each of the three word-problem units focuses on one word problem type and, after the first problem-type unit, subsequent units provide systematic, mixed cumulative review that includes previously taught problem types. The word problem types are Total (two or more amounts being combined), Difference (two amounts being compared), and Change (initial amount that increases or decreases). Each word-problem session comprises six activities. The first is the counting strategies review and flash card warm-up already described.

Word-problem warm-up, the next activity, lasts approximately 2 min and is initiated during the first word-problem unit. The tutor shows the student the word problem that the student had solved during the previous day's paper-and-pencil review. The student explains to the tutor how he or she solved the problem.

Conceptual and strategic instruction is the next activity. It lasts 15–20 min Tutors provide scaffolded instruction in the underlying structure of and in solving the three types of word problems (i.e., developing a schema for each problem type), along with instruction on identifying and integrating transfer features (to broaden students' schema for each problem type), using role-playing, manipulatives, instructional posters, modeling, and guided practice. In each lesson, students solve three word problems, with decreasing amounts of support from the tutor.

In the Total unit, the first problem type covered, tutors teach students to RUN through a problem: a 3-step strategy prompting students to Read the problem, Underline the question, and Name the problem type. Students used the RUN strategy across all three problem types. Next, for each problem type (i.e., schema), students are taught an algebraic equation to represent the underlying structure of that problem type and to identify and circle the relevant information that fills the slots of that equation. For example, for Total problems, students circle the item being combined and the numerical values representing that item, and then label the circled numerical values as "P1" (i.e., for part one), "P2" (i.e., for part two), and "T" (i.e., for the combined total). Students mark the missing information with an "X" and construct an algebraic equation representing the underlying mathematical structure of the problem type. For Total problems, the algebraic equation takes the form of "P1 + P2 = T;" and the "X" can appear in any of the three variable positions. Students are taught to solve for X, to provide a word label for the answer, and to check the reasonableness and accuracy of work. The strategy for Difference problems and Change problems follows similar steps but uses variables and equations specific to those problem types. For Difference problems, students are taught to look for the bigger amount (labeled "B"), the smaller amount (labeled "s"), and the difference between amounts (labeled "D"), and to use the algebraic equation "B – s = D." For Change problems, students are taught to locate the starting amount (labeled "St"), the changed amount (labeled "C"), and the ending amount (labeled "E"); the algebraic equation for Change problems is "St +/– C = E" (+/– depends on whether the change is an increase or decrease in amount).

For each problem type, explicit instruction to broaden schemas occurs in six ways. First, students are taught that because not all numerical values in word problems are relevant for finding solutions, they should identify and cross out irrelevant information as they identify the problem type. Second, students are taught to recognize and solve word problems with the missing information in the first or second position of the algebraic equation that represents the underlying structure of the problem type. Third, students learn to apply the problem-solving strategies to word problems that involve addition and subtraction with double-digit numbers with and without regrouping. Fourth, students learn to solve problems involving money. Fifth, students are taught to find relevant information for solving word problems in pictographs, bar charts, and pictures. Finally, students learn to solve 2-step problems that involve two problems of the same problem type or that combine problem types. Across the three problem-type units, previously taught problem types are included for review and practice.

Sorting word problems is the next activity. Tutors read aloud flash cards, each displaying a word problem. The student identifies the word problem type, placing the card on a mat with four boxes labeled "Total," "Difference," "Change," or "?." Students do not solve word problems; they sort them by problem type. To discourage students from associating a cover story with a problem type, the cards use similar cover stories with varied numbers, actions, and placement of missing information. After 2 min, the tutor notes the number of correctly sorted cards and provides corrective feedback for up to three errors.

In *paper-and-pencil review*, the final activity, students have 2 min to complete nine number sentences asking the student to find X. Then, students have 2 min to complete one word problem. Tutors provide corrective feedback and note the number of correct problems on the paper. Tutors require students to count up arithmetic errors, and keep the paper-and-pencil review sheet for the next day's *word-problem warm-up* activity.

A systematic *reinforcement* program is incorporated. Throughout each Pirate Math session, tutors award gold coins following each activity, with the option to withhold coins for inattention or poor effort. Throughout the session, each gold coin earned is placed on a "Treasure Map." Sixteen coins lead to a picture of a treasure box and, when reached, the student chooses a small prize from a real treasure box. The student keeps the old Treasure Map and receives a new map in the next lesson.

How Pirate Math addresses the first seventh principle of effective remediation. As shown in a series of field-based randomized control trials (Fuchs, Powell, Seethaler, et al., 2009; Fuchs, Powell, Seethaler, et al., 2010; Fuchs, Seethaler, et al., 2008), Pirate Math results in statistically significant and practically important effects on arithmetic fluency and word problems, even as it promotes better performance on procedural calculations and algebra. So Pirate Math is demonstrably efficacious. Nevertheless, as noted, no instructional method, even those validated using randomized control studies, works for all students. This makes it necessary to incorporate *ongoing progress monitoring* as an essential element of intensive remedial programming. Teachers use progress monitoring to determine whether a validated treatment protocol is in fact effective for a given student. When progress monitoring reveals that a student is failing to respond as expected to a validated intervention protocol, progress monitoring is then used to formulate an individually tailored instructional program that is in fact effective for that student.

Curriculum-based measurement (CBM) is the form of progress monitoring for which the preponderance of research has been conducted. To illustrate how CBM is used, consider the case of Francisco, a hypothetical student, who developed sizeable math deficits over the course of first and second grade, despite strong general education programming and even though small-group tutoring was implemented during the spring semester of second grade. At the beginning of third grade, Francisco was identified for remedial intervention. Mrs. LaBelle, the special education teacher, set Francisco's mathematics goal for year-end performance as competent second-grade performance. Relying on established methods, Mrs. LaBelle identified enough CBM tests to assess Francisco's performance each week across the school year. Each test systematically samples the second-grade mathematics curriculum in the same way, is administered in the same way, and is of equivalent difficulty. Each weekly score is an indicator of mathematics competence at the second grade. At the beginning of the year, she expected Francisco's performance to be low but as she addressed the curriculum over the school year, she expected his scores to gradually increase. Because each progress-monitoring test collected across the school year is of equivalent difficulty, each week's scores can be graphed and directly compared to each other. Also, a slope can be calculated on the series of scores. This slope quantifies Francisco's rate of improvement in terms of the weekly increase in score. In addition, because each week's assessment samples the annual curriculum in the same way, Mrs. LaBelle can derive a systematic analysis of which skills Francisco has and has not mastered at any point in time, and Mrs. LaBelle can look across time at a given skill to determine how Francisco's mastery has changed.

A large body of work indicates that CBM progress monitoring enhances teachers' capacity to plan mathematics programs and to effect stronger mathematics achievement among students with serious learning problems (Fuchs & Fuchs, 1998). To inform instructional planning, teachers rely on the CBM graphed scores. Once the teacher sets the year-end goal, the teacher draws the desired score on the graph at the date corresponding to the end of the year. The teacher then draws a straight line connecting the student's beginning-of-the-year score with the year-end goal. This line is called the goal line. It represents the approximate rate of weekly improvement (or slope) teachers hope a student will achieve. When a student's trend line (i.e., the slope through the student's actual scores) is steeper than the goal line, the teacher increases the goal for the student's year-end performance. When a student's trend line is flatter than the goal line, the teacher relies on her knowledge about the student along with a CBM analysis of the student's skills, derived from the CBM data, to revise the instructional program in an attempt to boost the weekly rate of student learning. Research shows that with CBM decision rules, teachers design more varied instructional programs that are more responsive to individual needs (Fuchs, Fuchs, & Hamlett, 1989b), that incorporate more ambitious student goals (Fuchs, Fuchs, & Hamlett, 1989a), and that result in stronger end-of-year scores on commercial, standardized tests (e.g., Fuchs et al., 1989a; Fuchs, Fuchs, Hamlett, & Stecker, 1991).

When Mrs. LaBelle assumed responsibility for Francisco's remediation program, she decided to use Pirate Math. This entailed tutoring for 30 min per session, three times per week. As Mrs. LaBelle began to implement this validated protocol, she also began to administer the CBM tests once each week for computation and once each week for concepts/applications. Mrs. LaBelle calculated Francisco's baseline or beginning-of-the-year performance, the median of his first three scores. Using CBM guidelines for goal setting, she decided that Francisco's year-end goal would require a weekly increase of .5 digits for computation and a weekly increase of .6 points for concepts/applications. So 25 weeks later, at the end of the school year, Francisco's year-end goal would be 18 digits correct on CBM computation and 18 points correct on CBM concepts/applications. Ten weeks later, Mrs. LaBelle compared lines of best fit through Francisco's actual CBM scores; calculated the slope of his actual improvement; and compared the slope against the desired rates of improvement (a weekly increase of .5 digits for computation and a weekly increase of .6 points for concepts/applications).

The CBM data showed that Pirate Math, with its focus on number combinations and procedural calculations, was producing strong growth for Francisco: His actual rate of improvement was steeper than the goal line. By contrast, Francisco was proving insufficiently unresponsive to Pirate Math's word-problem instruction, in which his actual rate of improvement was dramatically less steep than the goal line. Therefore, Mrs. LaBelle modified the Pirate Math standard protocol. She considered Francisco's performance during tutoring sessions and reviewed his performance on the CBM concepts/applications story problems. She determined that he was having difficulty differentiating problem types when irrelevant information was included in problems and when the missing information in problems occurred anywhere but the final position in the number sentence. Based on this analysis, Mrs. LaBelle added instruction on mixed problem types, lengthened the problem-type sorting activity, and added instructional time on irrelevant information and deriving number sentences when the missing information is in the first or second slot of the equation. As she implemented this revision in the intervention protocol, Mrs. LaBelle continued to monitor Francisco's responsiveness using weekly CBM. His learning improved; his slope grew steeper than the goal line. Teachers can use CBM in this formative, inductive, and recursive way to derive individual instructional programs that are effective for individual students and increase the probability of improved student outcomes.

Salient Issues Concerning MD and Its Remediation

In this section, we discuss three issues concerning MD and it remediation. The first issue is whether difficulty with arithmetic represents a bottleneck for successful performance with other mathematics skills and for students with MD (e.g., Fleishner et al., 1982; Geary et al., 1987; Goldman et al., 1988). The hypothesis is that, with a fixed amount of attention, students with arithmetic deficits allocate available resources for deriving answers to these simple problems instead of focusing on the more complex mathematics into which the arithmetic is embedded (cf. Ackerman, Anhalt, & Dykman, 1986; Goldman & Pellegrino, 1987). If arithmetic represents a bottleneck deficit, performance on more complex mathematics tasks should improve simply as a function of remediating arithmetic deficits, just as decoding intervention has been shown to improve reading comprehension (Blachman et al., 2004; Torgesen et al., 2001). An alternative perspective exists in the mathematics education literature that challenges the assumption of such vertical transfer, whereby mastery of simple skills facilitates acquisition of more complex skills (Gagne, 1968; Resnick & Resnick, 1992).

Few researchers have examined whether remediation of arithmetic deficits transfers to more complex math skills. Research conducted by Fuchs et al., which systematically assesses this issue, suggests that transfer may occur to some but not all aspects of mathematical performance. In some studies (Fuchs, Powell, Seethaler, et al., 2009; Fuchs, Powell, Seethaler, et al., 2010), we found support for this "bottleneck" hypothesis in the transfer we observed from arithmetic remediation to procedural calculation outcomes. And evidence for transfer was not entirely consistent (see Fuchs, Powell, et al., 2008; Powell et al., 2009). Moreover, we found no evidence in any study to support the bottleneck hypothesis on word-problem outcomes. With arithmetic improvement (but in the absence of word-problem tutoring), students with MD evidenced no improvement in solving word problems. This suggests that the source of their difficulty is not diverting attention from the complex mathematics to the arithmetic embedded in those problems, but rather failing to comprehend the relations among the numbers embedded in the narratives or to process the language in those stories adequately. Thus, arithmetic does not appear to be the bottleneck for word-problem performance. Instead, MD may represent a more complicated pattern of difficulty, implicating language as has been suggested elsewhere (e.g., Fuchs et al., 2005, 2006). Given these contradictory findings about transfer, in which some evidence supports transfer from arithmetic remediation to procedural calculations but no study has shown transfer to word problems, future work should continue to explore this issue.

The second issue concerns subtyping of MD. Because a key deficit associated with reading difficulty is phonological processing (Bruck, 1992) and because phonological processing deficits are linked to difficulty with automatic retrieval of math facts (Fuchs et al., 2005), students with concurrent difficulty in mathematics and reading (MDRD) should experience greater difficulty with arithmetic compared to students who experience difficulty only with mathematics (MD-only; Geary, 1993). Some research suggests that compared to students with MDRD, those with MD-only use more efficient counting procedures to solve arithmetic problems (Geary, Hamson, & Hoard, 2000; Jordan & Hanich, 2000) with faster retrieval times (Andersson & Lyxell, 2007; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Montani, 1997) but with comparable accuracy (Cirino et al., 2007). However, the literature is not consistent (e.g., Micallef & Prior, 2004; Reikeras, 2006), and most studies have employed a cross-sectional causal-comparative design.

An alternative approach is experimental, whereby students are stratified as MDRD versus MD-only and then randomly assigned to treatment or control conditions. The goal is to determine whether the subtypes respond differentially to intervention. This design offers the basis for stronger, causal inferences about the tenability of the subtyping scheme. In each of four studies, we adopted this approach. We found limited support for the MD-only versus MDRD framework for subtyping MD. The only evidence we found of differential responsiveness was Powell et al. (2009). MD-only students responded nicely to both arithmetic remediation conditions (practice remediation and conceptual remediation), with effect sizes for both remediation conditions around one standard deviation. By contrast, MDRD students proved unresponsive, with effect sizes near zero for both remediations. It is difficult to explain why MDRD students were differentially unresponsive in Powell et al., but not in the other three studies. In fact, the Powell et al. practice remediation was identical to the Fuchs et al. (2008) drill and practice remediation (which proved comparably efficacious for students with MD-only and MDRD). Moreover, the Fuchs, Powell, Seethaler, Cirino, Fletcher, Fuchs, Hamlett, and Zumeta (2009) counting strategies remediation was designed to circumvent the need for automatic retrieval, the hypothesized bottleneck for MDRD students. For that reason, we had hypothesized that the MDRD students might prove differentially responsive to the counting-up strategies than to the practice remediation; however, the response to both conditions was similar.

One explanation for the differential unresponsiveness in Powell et al. (2009) may involve the nature of the sample. Although students were screened in the same way for inclusion in all four studies, IQ for the Powell et al. (2009) MDRD sample scored, on average, 13 standard score points lower relative to their MD-only counterparts; by contrast, the IQ difference between MD-only and MDRD in each of the other studies was 7 points. When we incorporated IQ as a covariate in the Powell et al. analyses, findings remained similar. Even so, the lower IQ may explain the differentially poor response in Powell et al. or a third variable may explain the Powell et al. findings of MDRD students' low IQ scores as well as their poor response. Future work should explore the demographic and child characteristics associated with poor response to arithmetic remediation generally and in the context of the MDRD versus MD subtyping framework. Such research may provide important information about what drives arithmetic deficits and the nature of MD. It may also prove useful for guiding future intervention work. In the meantime, our line of experimental studies does not lend support to the MDRD/MD subtyping framework.

Our final issue concerns the treatment of MD. As already noted, despite the statistically significant and practically important effects associated with some remediation efforts, practitioners must always be mindful of individual response. That is, validated protocols will not work for all students, and schools therefore need to systematically monitor the effects of those validated remediations on individuals' learning and, when a validated protocol proves insufficiently effective, use the resulting data to tailor individualized programs. But the question remains: Once we determine, via ongoing progress monitoring, that a standard, validated remediation is not working, how might individual tailoring proceed?

One possibility is a skills-based diagnostic-prescriptive approach. For nonresponders, at the beginning of remediation, assessment might be conducted to determine the strategies with which a student derives answers to arithmetic problems (e.g., Siegler & Shrager, 1984). Then, using a menu of remediations developed to promote automatic retrieval with drill and practice versus to help students become fluent with counting strategies versus to build conceptual knowledge underlying math facts, the tutor might match the remediation approach to the student's profile of strategies. For example, if the assessment indicates that Wendy primarily relies on the immature total counting strategy to derive answers, the counting strategies remediation, with its focus on the more efficient counting strategies, might prove useful. Once Wendy consistently applies the min counting strategy with accuracy and fluency, the tutor might begin implementing conceptual lessons. After decomposition strategies associated with conceptual lessons are firm, the tutor might introduce intensive computerized practice. By contrast, let's say that Robert's strategy assessment reveals strong understanding of back-up (min counting as well as decomposition) strategies, but he nevertheless demonstrates an absence of automatic retrieval. For Robert, the tutor might intensify the repeated flash card activity, whereby students correct errors using back-up strategies they have mastered efficiently, trying to beat previous scores (as in repeated reading) with correct and fluent responding. Furthermore, the tutor might systematically mix the repeated flash card activity with computerized drill and practice, requiring Robert to apply his back-up strategies. And so on. A variation on this individualized approach was suggested by Goldman et al. (1988) when they documented clusters of students with different strategy patterns. Yet, to our knowledge, no research on its efficacy has been conducted. Experimental studies are needed to contrast such a diagnostic-prescriptive remediation against a standard protocol.

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