Math 25 Worksheet 15 - Taylor Poly's 2

1. Find the Taylor polynomial of degree 4 approximating the function $f(x) = e^{2x}$ for x near 0.

(we work "from scratch" as would have made sense when this worksheet came out.)

n	$f^{(n)}(x)$	at 0
0	e^{2x}	1
1	$2e^{2x}$	2
2	$2^2 e^{2x}$	2^2
3	$2^3 e^{2x}$	2^3
4	$2^4 e^{2x}$	2^4

So $P_4(x)$

$$= 1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + \frac{2^4x^4}{4!}$$

2. How does your polynomial above relate to the Taylor polynomial of degree 4 for e^x?

We see that $P_4(x)$ for e^{2x} is what we get when we replace x with $2x$ in 4^{th} degree Taylor poly for e^x:

$$1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!}$$

3. Find the Taylor polynomial of degree 4 approximating the function $f(x) = cos(3x)$ for x near 0. Use the short cut you observed in the previous problem?

We know $P_4(x)$ for $\cos x$ is $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

So $P_4(x)$ for $\cos(3x)$ is $1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!}$

4. Use the Taylor approximation for x near 0, $sin x \approx x - \frac{x^3}{3!}$, to explain why $\lim_{x\to 0} \frac{sin x}{x} = 1$. (This is #29 from §10.1.)

We have $\sin x \approx x - \frac{x^3}{3!}$ for x near 0.

So $$\frac{\sin x}{x} \approx \frac{x - \frac{x^3}{3!}}{x} = 1 - \frac{x^2}{3!}.$$

Taking limit as $x \to 0$, the approximation becomes exact, so

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} 1 - \overset{\to 0}{\frac{x^2}{3!}} = 1.$$