Math 25 Substitution Spring 2016
(worksheet 2)

1. $\int (1+x^2)^{1/3} 2x\,dx = \int w^{1/3}\,dw = \frac{w^{4/3}}{4/3} + C$

Let $w = 1+x^2$, $dw = 2x\,dx$

$= \frac{3}{4}(1+x^2)^{4/3} + C$

2. $\int_2^3 \frac{1}{x \ln x}\,dx$. We see $\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}\frac{1}{x}dx = \int \frac{1}{w}dw$

$= \ln|w| + C$.

Let $w = \ln x$, $dw = \frac{1}{x}dx$

So original integral equals $\ln|\ln x|\Big]_2^3$

$= \ln 3 - \ln 2$.

3. $\int \frac{1}{1+4x^2}dx = \int \frac{1}{1+w^2}\frac{1}{2}dw = \frac{1}{2}\arctan w + C$

$= \frac{1}{2}\arctan(2x) + C$

Let $w = 2x$

$dw = 2dx$

$\frac{1}{2}dw = dx$

4. $\int_0^{\pi/3} \frac{\sin\theta}{\cos^2\theta}\,d\theta = \int_{\theta=0}^{\theta=\pi/3} \frac{1}{w^2}(-dw) = -\int_{\theta=0}^{\theta=\pi/3} w^{-2}\,dw = -\frac{w^{-1}}{-1}\Big]_{\theta=0}^{\theta=\pi/3}$

$= \frac{1}{\cos\theta}\Big]_0^{\pi/3} = \frac{1}{1/2} - \frac{1}{1} = 2-1 = 1.$

Let $w = \cos\theta$

$dw = -\sin\theta\,d\theta$

$-dw = \sin\theta\,d\theta$

5. $\int \frac{x^2}{\sqrt{1-x}}dx = \int \frac{(1-w)^2}{\sqrt{w}}(-dw) = -\int \frac{1-2w+w^2}{w^{1/2}}dw$

$= -\int w^{-1/2} - 2w^{1/2} + w^{3/2}\,dw$

$= -\left(\frac{w^{1/2}}{1/2} - \frac{2w^{3/2}}{3/2} + \frac{w^{5/2}}{5/2}\right) + C$

$= -\left(2(1-x)^{1/2} - \frac{4}{3}(1-x)^{3/2} + \frac{2}{5}(1-x)^{5/2}\right) + C$

Let $w = 1-x$, so $x = 1-w$, and $dw = -dx$, $-dw = dx$,

6. $\int \overbrace{\sin^4(3\theta)}^{w^4} \overbrace{\cos 3\theta \, d\theta}^{\frac{1}{3}dw} = \int w^4 \tfrac{1}{3} dw = \tfrac{1}{3}\int w^4 dw$

Let $w = \sin(3\theta)$

$dw = 3\cos 3\theta \, d\theta$

$\tfrac{1}{3} dw = \cos 3\theta \, d\theta$

$= \tfrac{1}{3}\tfrac{w^5}{5} + C = \tfrac{1}{15}(\sin 3\theta)^5 + C$

same as $\tfrac{1}{15}\sin^5(3\theta) + C$

7. $\int \frac{e^{1/t}}{t^2} dt = \int e^w (-dw) = -e^w + C = -e^{1/t} + C$

Let $w = 1/t$

$dw = -\frac{1}{t^2} dt$

8. $\int t\sqrt{t-1} \, dt = \int (w+1)\sqrt{w} \, dw = \int (w+1) w^{1/2} dw$

Let $w = t-1$

$dw = dt$

and $t = w+1$

$= \int w^{3/2} + w^{1/2} dw = \frac{w^{5/2}}{5/2} + \frac{w^{3/2}}{3/2} + C$

$= \tfrac{2}{5}(t-1)^{5/2} + \tfrac{2}{3}(t-1)^{3/2} + C$

9. $\int_1^e \frac{\sin(\ln x)\cos(\ln x)}{x} dx = \int_{x=1}^{x=e} w \, dw = \tfrac{1}{2}w^2 \Big]_{x=1}^{x=e}$

Let $w = \sin(\ln x)$

$dw = \cos(\ln x)\left(\frac{1}{x} dx\right)$

$= \tfrac{1}{2}\left[\sin(\ln x)\right]^2 \Big]_1^e$

$= \tfrac{1}{2}\left[(\sin(1))^2 - (\sin(0))^2\right] = \tfrac{1}{2}\sin^2 1$

Typo (sorry!!) should have been $\theta^2$, Yikes!

10. $\int \frac{\theta \sin\theta^2}{\sqrt{\cos(\theta^2)}} d\theta = -\frac{1}{2}\int \frac{1}{\sqrt{w}} dw = -\frac{1}{2}\frac{w^{1/2}}{1/2} + C$

Let $w = \cos(\theta^2)$

$dw = -\sin(\theta^2)(2\theta) d\theta$

$-\tfrac{1}{2} dw = \sin(\theta^2)\,\theta \, d\theta$

$= -\sqrt{\cos(\theta^2)} + C,$