Creativity and Economics

Designing an Optimal Contract Under Moral Hazard

There are many ways to think about contracts. In general, contracts are entered into so that parties to the contract have good motives to maximize the joint outcome of the activities covered by the contract. For example, an optimal apartment rental contract will be written so that both landlord and tenant retain an incentive to maintaining the apartment's condition as fit for living. Elements of the contract may not be fully spelled out, such as the landlord's future option to raise the rent, or the tenant's future desire to stay in or leave the apartment.

Moral hazard occurs when actions may be hidden from one of the contracting parties. The classic example is accident insurance; once an accident is insured against, the person insured may fail to take (hidden) actions that would prevent the accident. An extreme example is arson – if the insured property owner believes that the property is worth less than the insurance, he or she may torch the property to collect the insurance. If actions were not hideable, then a more complete contract would be written (such as that if the property owner commits arson, the insurance payment will not be made) -- the more complete contract would always be enforceable.

One consequence is that insurance is often made incomplete (insurance may not cover the full amount of the damage, or the first thousand dollars of damage), so that the insured party has some incentive to prevent the accident from occurring. The optimal contract balances the cost of incomplete insurance against the cost of bad incentives.

An optimal contract under moral hazard is efficient in the sense that no other contract is better for all parties (that is, makes one party strictly better off and no party worse off). The optimal contract is one which satisfies *participation constraints* based upon *individual rationality* (the individuals voluntarily participate in the contract because their alternatives are no better) and *incentive compatibility* (the actions taken under the contract offer the highest expected utility to the individual taking the action.)

Put another way, both the corporation and the worker must voluntarily enter into the contract (individual rationality); the worker must voluntarily do the work the contract requires (incentive compatibility); and the outcomes specified under the contract must be attainable (feasilibity).

In this section we solve a simplified moral hazard problem in which a worker has a project (a book, let's say) which if the worker works hard (e=1, where e is effort) the book will succeed and make a profit of 9 with probability 0.5 and fail with probability 0.5, but if the worker doesn't work hard (e=0) then the book has no chance of succeeding. The worker has square root utility, so the worker is risk averse, and effort, which can be only 1 or 0, has a direct utility cost equal to the amount of effort.

Example 1:
The worker has utility
$U = c^{.5} – e$

Effort of 1 raises the probability p of the project succeeding to 0.5:
p= .5 e, and
output is 9 with prob p and 0 with prob 1-p.

2 possible levels of effort:
high e = 1 or low e = 0

First we consider what happens when the worker works on her own. This is a case called autarchy (meaning self rule).

Case 1: Autarchy: the worker on her own
The worker on her own will choose high effort.
How do we know this? We compare high effort and low effort, and see which gives higher utility.

High effort, costs the worker 1 unit of effort (measured in units of utility). With high effort, the worker will either receive utility of 3 (sqrt of 9) or utility of 0, each with probability one-half.
Expected utility = .5(3) +.5(0) –1 = 0.5
With low effort, the worker gets an expected utility of 0. So the worker is better off choosing high effort.
Thus the worker on her own will choose high effort and earn expected utility of 0.5.

Case 2. No moral hazard, the worker insures herself with a risk-neutral corporation.
Suppose there was no moral hazard, and the worker could insure herself with a risk neutral corporation.

Now suppose the worker goes to a (risk neutral) corporation and offers to give the corporation the proceeds of her work in exchange for a work contract.
If the worker's effort could be observed, the corporation could offer the worker a contract that depended on the worker working hard. In this case, the corporation would offer the worker a payment which was independent of whether the book succeeded or not.
Since the worker's effort cannot be observed, the contract can only depend on the success of the book.
Terms of the contract:

1. Risk neutral corporation will agree to a contract which guarantees that the worker will work hard (incentive compatibility) and that the expected total payment to the worker will be less than or equal to the expected value of the worker's output (participation constraint or individual rationality).
2. The worker will agree to the contract as long as the worker's expected utility is at least as high as her expected utility working on her own (participation constraint or individual rationality).

If there is no solution that meets these constraints simultaneously, then there will be no contract, and autarchy will result. If there is more than one solution, then which solution is picked will depend on the bargaining power of the two sides – e.g. the worker may be the monopolist due to some unique skill possessed by the worker.

The contract offers: w_h if the book succeeds and w_l if not. The success of the book is observable and can be contracted upon, while the author's effort cannot be contracted upon (or so we assume in this problem).

If the worker works hard, the worker gets:
$.5\, w_h^{.5} + .5 w_l^{.5} - 1$
and if the worker doesn't work hard, the worker gets
$(w_l^{.5})$
3. The only advantage to the corporation is if the worker works hard, so
Incentive compatibility must hold (the worker must prefer working hard to not working)
$.5\, w_h^{.5} + .5 w_l^{.5} - 1 > w_l^{.5}$
Or
$w_h^{.5} > w_l^{.5} + 2$
Also, the worker must do at least as well as she would do on her own, so (individual rationality):
$.5\, w_h^{.5} + .5 w_l^{.5} - 1 > 0.5$
or
$w_h^{.5} + w_l^{.5} > 3$
and the corporation must make an expected profit of at least zero, so the expected reward (9 with probability .5) must exceed expected wage payments ($.5\, w_h + .5\, w_l$)
$.5\,(9) > .5\, w_h + .5\, w_l$
or
$9 > w_h + w_l$

Solution one:
The corporation's preferred contract
Make the incentive compatibility condition an equation, and the worker's individual rationality constraint also an equation.
$w_h^{.5} = w_l^{.5} + 2$ (incentive compatibility)
$w_h^{.5} + w_l^{.5} = 3$ (worker's individual rationality)
Can solve this algebraically.
$w_h = 6.25$, $w_l = 0.25$
Corporation gets 4.5 – 3.25 = 1.25

Solution two:
The worker's preferred contract:
Make the incentive compatibility condition an equation, and the corporation's individual rationality constraint also an equation.
$w_h^{.5} = w_l^{.5} + 1$ (incentive compatibility)
$w_h + w_l = 9$ (corporation's individual rationality)
Use numerical approximation to solve. $w_h = 8.2416$, $w_l = 0.7583$

This table presents alternative wage scenarios.

	wages		utility of wages		utility of effort		Corp	Individually	Incentive
Wage alternatives	w(9)	w(0)	sqrt(w(9))	sqrt(w(0)	ut(eh)	ut(el)	Prof(eh)	rational?	compatible
	4	0	2	0	0.00	0.00	2.50	No	Yes
	5	0.0557	2.23607	0.2361	0.24	0.24	1.97	No	Yes
	6	0.202	2.44949	0.4495	0.45	0.45	1.40	No	Yes
best for corporation	6.25	0.25	2.5	0.5	0.50	0.50	1.25	Yes	Yes
	7	0.417	2.64575	0.6458	0.65	0.65	0.79	Yes	Yes
best for creative worker	8.2416	0.7583	2.87082	0.8708	0.87	0.87	0.00	Yes	Yes
autarchy	9	0	3	0	0.50	0.00	0.00	Yes	Yes
with monitoring									
first best for creative wor	4.5	4.5	2.12132	2.1213	1.12	2.12	0.00	Yes	No
	3.5	3.5	1.87083	1.8708	0.87	1.87	1.00	Yes	No
first best for corporation	2.25	2.25	1.5	1.5	0.50	1.50	2.25	Yes	No

Example 2:
A worker has utility
$U = c^{.5} – e$
Output is 9 with prob p and 0 with prob 1-p
Probability p is determined by effort:
$p=e – e^2/2$

$(e – e^2)$
e = ½
p = ¼
value = 1
exp ut = ¼

e = .75
p = .46875
value = 1.875
1.37-.75 =.62
e,f are efforts

Example 3: Can workers share output (moral hazard)
Worker 1 has utility
U(1) = c – e
Worker 2 has utility
U(2) = c - f

Production
is:
$v(e-e^2/2+f-f^2/2)$
where v is random and cannot be observed, exp value is 4.
Moreover, e and f cannot be observed except by the worker who does the work.

Then
assuming workers share output, consumption is
$v((e-e^2/2+f-f^2/2)/2$
Max utility at (v-ve)-2=0
e= (v-2)/v
e,f =1/2
Exp out = 3
Exp c = 1.5
U=1

But optimality is e=(v-1)/v=.75
Exp out =3.5
Exp c =1.875
U = 1.125

Because of sharing, and some of the output goes to the partner, then there is some shirking.
Solutions:
monitoring
reputation